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SOME NEW FIXED-POINT THEOREM BY APPLYING INTIMATE MAPPING IN INTUITIONISTIC FUZZY METRIC SPACE

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Abstract: The objective of this paper is to demonstrate common fixed point theorem using rational contractive mapping by deploy intimate mapping in Intuitionistic fuzzy metric space. Our outcomes stretch few known outcomes which arises in the literature.

Keywords - Intimate mapping, Compatible mapping of type (A), Intuitionistic fuzzy metric space.

1. Introduction: In 1905 French Mathematician Maurice Frechet & accurately in 1906 during his doctoral thesis he studied distance functional $d: X \times X \to R$ within any of two given elements x, y of a set X so that it fulfills four postulates $\forall$ x, y, z $\in X$

(i) $d(x, y) \geq 0$

(ii) $d(x, y) = 0 \; iff \; x = y$

(iii) $d(x, y) = d(y, x)$

(iv) $d(x, y) \leq d(x, z) + d(z, y)$

During the year 1965, fuzzy set as a notion was brought by Zadeh & as defined as "A fuzzy set A in X is a function with domain X & values in [0,1]"

After a decade fuzzy metric was conceptualized by Kramosil & Michalek [9] in 1975. Fuzzy metric space as a concept is updated by George & Veeramani by applying continuous t-norm. The concept of fuzzy metric space was taken up by various authors in number of ways. As a generalization of fuzzy set, Intuitionistic fuzzy set was introduced by K. Atanassov [1] in 1988.

Let X be a non-empty fixed set. An intuitionistic fuzzy set A is an object having the form

$$A = \{< x, (\mu_A(x), v_A(x) >: x \in X\}$$

Where the function $\mu_A(x) \to [0,1]$ and $v_A(x) \to [0,1]$ denote the degree of membership and the degree of nonmembership of each element $x \in X$ to the set A respectively and

$$0 \leq \mu_A(x) + v_A(x) \leq 1 \text{ for each } x \in X$$

The completeness of the fuzzy metric space (G-complete fuzzy metric space) is defined by Grabiec[4] and extended the Banach Contraction theorem to G-complete fuzzy metric spaces. In 1997, C,oker introduced the concept of the so called "Intuitionistic fuzzy topological spaces". It defines the notion of Intuionistic fuzzy topological spaces.

$$0_{\sim} = \{< x, 0,1 >: x \in X\} \; 1_{\sim} = \{< x, 1,0 >: x \in X\}$$

An intuitionistic fuzzy topology on a nonempty set X is a family τ of intuitionistic fuzzy sets in X satisfy the following axioms:

(T_1) $0_{\sim}, 1_{\sim} \in \tau$;

(T_2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$;

(T_3) $\cup G_i \in \tau$ for any arbitrary family $\{G_i : i \in J\} \subseteq \tau$

In this case, the pair (X, τ) is called an intuitionistic fuzzy topological space.

By using the concept of t-norm and t-conorm which is established by Schweizer and Skalar in 1960, Park introduced the notion of intuitionistic fuzzy metric spaces [13] in 2004. After that many authors studied this concept and generated fixed point theorems in intuitionistic fuzzy metric space. In 2006 Sadati and Park came with new idea of intuitionistic fuzzy normed spaces[16]

Intimate mapping have been inducted by Sahu et al. [15]. As a matter of fact that intimate mapping is the extension of the compatible mapping of type (A) & as a key type of mapping was introduced by Kang & Kim [8]. The fascinating attribute of compatible mapping of type (A), weakly compatible maps & weekly compatible mappings of type (A) is that all prementioned mapping commute at coincidence points. On the other hand intimate maps do not compulsorily commute at coincidence point.

Method: The paper is organised as follows. Section 2 contains the preliminaries, including definitions and lemmas with corresponding references that will be used in sequel. Section 3 contains the main result of the paper.

2. Preliminaries

Definition 2.1 A binary operation $*: [0,1]^2 \to [0,1]$ is called a continuous triangular norm (shortly t-norm) if it satisfies the following conditions:

(i) $*$ is associative & commutative.
(ii) $*$ is continuous.
(iii) $a * 1 = a$ $\quad$ a $\in [0,1]$
(iv) $a * b \le c * d$ $\quad$ whenever $a \le c$ & $b \le a$ $\forall\, a, b, c, d \in [0,1]$

Definition 2.2 A binary operation $\diamond: [0,1]^2 \to [0,1]$ is a continuous triangular conorm (t-conorm) if $\diamond$ satisfies the following conditions:

(i) $\diamond$ is commutative & associative.
(ii) $\diamond$ is continuous.
(iii) $a \diamond 0 = 0$ $\quad \forall$ a $\in [0,1]$
(iv) a$\diamond$ b $\le$ c $\diamond$ d whenever $a \le c$ & $b \le d$ & $a, b, c, d \in [0,1]$

Definition 2.3 A 5-tuple $(X, \mathbb{M}, \mathbb{N}, *, \diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ a continuous t-norm, $\diamond$ a continuous t-conorm & $\mathbb{M}$, $\mathbb{N}$ are fuzzy set on $X^2 \times [0, \infty)$ satisfying the following conditions $\forall\, x, y, z \in X$ $\quad$ s, t > 0

(i) $\mathbb{M}(x, y, t) + \mathbb{N}(x, y, t) \le 1$
(ii) $\mathbb{M}(x, y, 0) = 0$
(iii) $\mathbb{M}(x, y, t) = 1 \quad \forall\, t > 0 \text{ iff } x = y$
(iv) $\mathbb{M}(x, y, t) = \mathbb{M}(y, x, t)$
(v) $\mathbb{M}(x, y, t) + \mathbb{M}(y, z, s) \le \mathbb{M}(x, z, t + s)\ \forall\, x, y, z \in X\ \ s, t > 0$
(vi) $\mathbb{M}(x, y, \cdot)$: *[0,∞]* $\to [0,1]$ *is left continuous.*
(vii) $\lim_{t\to\infty} \mathbb{M}(x, y, t) = 1 \quad \forall\, x, y \in X$
(viii) $\mathbb{N}(x, y, 0) = 1$
(ix) $\mathbb{N}(x, y, t) = 0 \ \ \forall\, t > 0 \ \text{ iff } x = y$
(x) $\mathbb{N}(x, y, t) = \mathbb{N}(y, x, t)$
(xi) $\mathbb{N}(x, y, t) \diamond \mathbb{N}(y, z, s) \ge \mathbb{N}(x, z, t + s)\ \forall\, x, y, z \in X\ \ s, t > 0$
(xii) $\mathbb{N}(x, y, \cdot): [0, \infty) \to [0,1]$ *is right continuous;*
(xiii) $\lim_{t\to\infty} \mathbb{N}(x, y, t) = 0 \quad \forall x, y \in X$

Then $(\mathbb{M}, \mathbb{N})$ is called an Intuitionistic Fuzzy Metric on X. The functions $\mathbb{M}(x, y, t)$ & $\mathbb{N}(x, y, t)$ denote a degree of nearness & a degree of non nearness between x & y.

Definition 2.4 let $(X, \mathbb{M}, \mathbb{N}, *, \diamond)$ is IFMS & $\{x_n\}$ is said to be Cauchy sequence if $\lim_{n\to\infty} \mathbb{M}(x_{n+r}, x_n, t)$=1 & $\lim_{n\to\infty} \mathbb{N}(x_{n+r}, x_n, t) = 0$ for every $t > 0$ and $r > 0$.

Definition 2.5 let $(X, \mathbb{M}, \mathbb{N}, *, \diamond)$ is IFMS & $\{x_n\}$ is a convergence sequence if it is convergent to x $\in X$ if $\lim_{n\to\infty} \mathbb{M}(x_n, x, t) = 1$ & $\lim_{n\to\infty} \mathbb{N}(x_n, x, t) = 0$ for each $t > 0$.

Definition 2.6 An intuitionistic fuzzy metric space is called complete iff every Cauchy sequence in X converges in X.

Definition 2.7 Let f & g be two mapping of Intuitionistic fuzzy metric space $(X, \mathbb{M}, \mathbb{N}, *, \diamond)$ into itself

(a) Compatible if
$\lim_{n\to\infty} \mathbb{M}(fgx_n, gfx_n, t) = 1$ & $\lim_{n\to\infty} \mathbb{N}(fgx_n, gfx_n, t) = 0$
Whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = w$ for some $t \in X$.

(b) Compatible of type (A)
$\lim_{n\to\infty} \mathbb{M}(fgx_n, gfx_n, t) = 1$ & $\lim_{n\to\infty} \mathbb{M}(gfx_n, fx_n, t) = 1$
$\lim_{n\to\infty} \mathbb{N}(fgx_n, gfx_n, t) = 0$ & $\lim_{n\to\infty} \mathbb{N}(gfx_n, fx_n, t) = 0$
Whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = w$

Definition 2.8 Let $(X, \mathbb{M}, \mathbb{N}, *, \diamond)$ be a intuitionistic fuzzy metric space. A point $x \in X$ is said to be limit point of $S \subset X$ iff $(B_\varepsilon(X) \setminus \{x\}) \cap S \ne \emptyset$ for every $\varepsilon > 1$ the set of all limit points of the set S is denoted by S

Definition 2.9 Let $(X, \mathbb{M}, \mathbb{N}, *, \diamond)$be a intuitionistic fuzzy metric space. We call a set S⊂ X closed in intuitionistic fuzzy metric space if S contains all of its limit points.

Lemma 2.10 Let $(X, \mathbb{M}, \mathbb{N}, *, \diamond)$ be a intuitionistic fuzzy metric space $\{x_n\}$ be a sequence in X and $x \in X$ then $\{x_n\}$ be a sequence in X and $x \in X$ then $x_n \to X$ $(n \to \infty)$ iff $\mathbb{M}(x_n, x, t) = 1$ and $\mathbb{N}(x_n, x, t) = 0$.

Theorem 2.11 Let $\{x_n\}$ be a Cauchy sequence in Intuitionistic fuzzy metric space $(X, \mathbb{M}, \mathbb{N}, *, \diamond)$ If the sequence $\{x_n\}$ has a subsequence $\{x_{nk}\}$ such that $\{x_{nk}\} \to x \in X$ $(n_k \to \infty)$ then $\{x_n\} \to X$ $(n \to \infty)$.

Definition 2.12 Let f & g be two mappings of a intuitionistic fuzzy metric space $(X, \mathbb{M}, \mathbb{N}, *, \diamond)$ into itself the f and g are said to be

(a) g −intimate mapping if
$\alpha M(gfx_n, gx_n, t) \geq \alpha M(ffx_n, fx_n, t)$ Where $\alpha = limsup$
$\beta N(gfx_n, gx_n, t) \leq \beta N(ffx_n, fx_n, t)$ Where $\beta = liminf$
$\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = p$ for some $p \in X$

(b) f − intimate mapping
$\alpha\mathbb{M}(fgx_n, fx_n, t) \geq \alpha\mathbb{M}(ggx_n, gx_n, t)$ Where $\alpha = limsup$
$\beta\mathbb{N}(fgx_n, fx_n, t) \leq \beta\mathbb{N}(ggx_n, gx_n, t)$ Where $\beta = liminf$
$\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$ for some $t \in X$

Proposition 2.13 Let f & g be mapping of a Intuitionistic fuzzy metric space $(X, \mathbb{M}, \mathbb{N}, *, \diamond)$ Assume that f & g are compatible of type (A) then f & g are f −intimate and g −intimate.

Proposition 2.14 Let f & g be mapping of a Intuitionistic fuzzy metric space $(X, \mathbb{M}, \mathbb{N}, *, \diamond)$ into itself. Assume that f & g are g −intimate & $fw = gw = p \in X$ Then
$\mathbb{M}(gp, p, t) \geq \mathbb{M}(fp, p, t)$
$\mathbb{N}(gp, p, t) \leq \mathbb{N}(gp, p, t)$

Proof: Suppose that $\lim_{n\to\infty} x_n = w \quad \forall\, n \geq 1$

$$fx_n = gx_n \to fw = gw = p$$

Since f & g are g −intimate, we have

$$\mathbb{M}(gfw, gw, t) = \lim_{n\to\infty} \mathbb{M}(gfx_n, gx_n, t) \leq \lim_{n\to\infty} \mathbb{M}(ffx_n, fx_n, t) = \mathbb{M}(ffw, fw, t)$$
$$\Rightarrow \mathbb{M}(gp, p, t) \leq \mathbb{M}(fp, p, t)$$
$$\mathbb{N}(gfw, gw, t) = \lim_{n\to\infty} \mathbb{N}(gfx_n, gx_n, t) \geq \lim_{n\to\infty} \mathbb{N}(ffx_n, fx_n, t) = \mathbb{N}(ffw, fw, t)$$
$$\Rightarrow \mathbb{N}(gp, p, t) \leq \mathbb{N}(fp, p, t)$$

3. Main Result

Theorem 3.1: Let P, Q, S, T be continuous mapping such that $S(X) \subseteq P(X)$ & $T(X) \subseteq Q(X)$ of complete intuitionistic fuzzy metric space $(X, \mathbb{M}, \mathbb{N}, *, \diamond)$ with t-norm & t-conorm

(a) $\mathbb{M}(Sx, Ty, t)$

$$\geq min\left\{\begin{matrix} \dfrac{2[\mathbb{M}(Qx, Sx, t) + \mathbb{M}(Py, Ty, t)]\mathbb{M}(Py, Sx, t)\mathbb{M}(Py, Ty, t)}{\big(1 + \mathbb{M}(Qx, Sx, t)\big)(\mathbb{M}(Py, Ty, t) + 1)}, \\ \{\mathbb{M}(Qx, Sx, t) \circledast \mathbb{M}(Py, Ty, t) \circledast \mathbb{M}(py, sx, t)\} \end{matrix}\right\}$$

(b) $\mathbb{N}(Sx, Ty, t)$

$$\leq max\left\{\begin{matrix} \dfrac{2[\mathbb{N}(Qx, Sx, t) + \mathbb{N}(Py, Ty, t)]\mathbb{N}(Px, Sx, t)\mathbb{N}(Py, Ty, t)}{\big(1 + \mathbb{N}(Qx, Sx, t)\big)(\mathbb{N}(Py, Ty, t) + 1)}, \\ \{\mathbb{N}(Qx, Sx, t) \boxdot \mathbb{N}(Py, Ty, t) \boxdot \mathbb{N}(py, sx, t)\} \end{matrix}\right\}$$

(c) Assume that $Q(x)$ is complete and the pairs Q, S is Q −intimate & P, T is P −intimate then Q, P, S &T have unique common fixed point.

(d) Pair Q, S & P, T is compatible of type A.

Proof: Let us consider any arbitrary point x_0 of X. As given $S(X) \subset P(X)$ hence there exist another x_1 of X in such a way that $Sx_0 = Px_1 = y_0$

Using the induction there exist a sequence $\{x_n\}$ and $\{y_n\}$ in X so that

$$Sx_{2n} = Px_{2n+1} = y_{2n}$$

$$T(X) \subset Q(X)$$

$$x_{2n+1} = Qx_{2n+2} = y_{2n+1}$$

Since Qx is complete there exist $a \in Qx$ such that $y_{2n+1} = Tx_{2n+1} = Qx_{2n+2} = a$ as $n \to \infty$

Now there exist $z \in X$ s.t. $Qz = a$

If y_n is a cauchy sequence then $\{y_{2n+1}\}$ and $\{y_{2n}\}$ are also convergent, therefore we have

$$y_{2n} = Sx_{2n} = Px_{2n+1} = a \text{ as } n \to \infty$$

Let's have an assertion that $Sz = a$, by equation (a) using replacement $x = z, y = x_{2n+1}$

$$\mathbb{M}(Sz, Tx_{2n+1}, kt) \geq min \begin{Bmatrix} \frac{2[\mathbb{M}(Qz,Sz,t)+\mathbb{M}(Px_{2n+1},Tx_{2n+1},t)]\mathbb{M}(Px_{2n+1},Sz,t)\mathbb{M}(Px_{2n+1},Tx_{2n+1},t)}{(1+\mathbb{M}(Qz,Sz,t))(\mathbb{M}(Pu_{2n+1},Tu_{2n+1},t)+1)}, \\ \{\mathbb{M}(Qz,Sz,t) \circledast \mathbb{M}(Px_{2n+1},Tx_{2n+1},t) \circledast \mathbb{M}(px_{2n+1},sz,t)\} \end{Bmatrix}$$

$$\mathbb{M}(Sz, a, t) \geq min \begin{Bmatrix} \frac{2[\mathbb{M}(a,Sz,t)+\mathbb{M}(a,a,t)]\mathbb{M}(a,Sz,t)\mathbb{M}(a,a,t)}{(1+\mathbb{M}(a,Sz,t))(\mathbb{M}(a,a,t)+1)}, \\ \{\mathbb{M}(a,Sz,t) \circledast \mathbb{M}(a,a,t) \circledast \mathbb{M}(a,sz,t)\} \end{Bmatrix}$$

$$\mathbb{M}(Sz, a, kt) \geq min \begin{Bmatrix} \frac{2[\mathbb{M}(a,Sz,t)+1]\mathbb{M}(a,Sz,t)}{(1+\mathbb{M}(a,Sz,t))(1+1)}, \\ \{\mathbb{M}(a,Sz,t) \circledast 1 \circledast \mathbb{M}(a,sz,t)\} \end{Bmatrix}$$

$$\mathbb{M}(Sz, a, kt) \geq min \begin{Bmatrix} \mathbb{M}(a,Sz,t), \\ \{\mathbb{M}(a,Sz,t) \circledast 1 \circledast \mathbb{M}(a,sz,t)\} \end{Bmatrix}$$

$$\mathbb{M}(Sz, a, kt) \geq \mathbb{M}(a, Sz, t)$$

which implies $Sz = a = Qz$

By using equation (b)

$$\mathbb{N}(Sz, Tx_{2n+1}, kt) \leq max \begin{Bmatrix} \frac{2[\mathbb{N}(Qz,Sz,t)+\mathbb{N}(Px_{2n+1},Tx_{2n+1},t)]\mathbb{N}(Px_{2n+1},Sz,t)\mathbb{N}(Px_{2n+1},Tx_{2n+1},t)}{(1+\mathbb{N}(Qz,Sz,t))(\mathbb{N}(Px_{2n+1},Tx_{2n+1},t)+1)}, \\ \{\mathbb{N}(Qz,Sz,t) \boxdot \mathbb{N}(Px_{2n+1},Tx_{2n+1},t) \boxdot \mathbb{N}(px_{2n+1},sz,t)\} \end{Bmatrix}$$

$$\mathbb{N}(Sz, a, kt) \leq max \begin{Bmatrix} \frac{2[\mathbb{N}(a,Sz,t)+\mathbb{N}(a,a,t)]\mathbb{N}(a,Sz,t)\mathbb{N}(a,a,t)}{(1+\mathbb{N}(a,Sz,t))(\mathbb{N}(a,a,t)+1)}, \\ \{\mathbb{N}(a,Sz,t) \boxdot \mathbb{N}(a,a,t) \boxdot \mathbb{N}(a,sz,t)\} \end{Bmatrix}$$

$$\mathbb{N}(Sz, a, kt) \leq max \begin{Bmatrix} 0, \\ \{\mathbb{N}(a,Sz,t) \boxdot 0 \boxdot \mathbb{N}(a,sz,t)\} \end{Bmatrix}$$

$$\mathbb{N}(Sz, a, kt) \leq \mathbb{N}(Pv, Tv, t)$$

which implies that $Sz = a$

Hence, we proved that $Sz = Qz = a$

Next, we will strongly believe that $Tr = a$, Also $Sz = a \in Sx \subset Px$

Therefore, there exist a point $r \in X$ in such a way that $Pr = a$

By the substitution as $x = z$ and $y = r$ in (a) and (b)

$$\mathbb{M}(Sz, Tr, kt) \geq min \begin{Bmatrix} \dfrac{2[\mathbb{M}(Qz, Sz, t) + \mathbb{M}(Pr, Tr, t)]\mathbb{M}(Pr, Sz, t)\mathbb{M}(Pr, Tr, t)}{(1 + \mathbb{M}(Qz, Sz, t))(\mathbb{M}(Pr, Tr, t) + 1)}, \\ \{\mathbb{M}(Qz, Sz, t) \circledast \mathbb{M}(Pr, Tr, t) \circledast \mathbb{M}(Pr, sz, t)\} \end{Bmatrix}$$

$$\mathbb{M}(a, Tr, kt) \geq min \begin{Bmatrix} \dfrac{2[\mathbb{M}(a, a, t) + \mathbb{M}(a, Tr, t)]\mathbb{M}(a, a, t)\mathbb{M}(a, Tr, t)}{(1 + \mathbb{M}(a, a, t))(\mathbb{M}(a, Tr, t) + 1)}, \\ \{\mathbb{M}(a, a, t) \circledast \mathbb{M}(a, Tr, t) \circledast \mathbb{M}(a, a, t)\} \end{Bmatrix}$$

$$\mathbb{M}(a, Tr, kt) \geq min \begin{Bmatrix} \dfrac{2[\mathbb{M}(a, a, t) + \mathbb{M}(a, Tr, t)]\mathbb{M}(a, a, t)\mathbb{M}(a, Tr, t)}{(1 + \mathbb{M}(a, a, t))(\mathbb{M}(a, Tr, t) + 1)}, \\ \{\mathbb{M}(a, a, t) \circledast \mathbb{M}(a, Tr, t) \circledast \mathbb{M}(a, a, t)\} \end{Bmatrix}$$

$$\mathbb{M}(a, Tr, kt) \geq min \begin{Bmatrix} \dfrac{2[1 + \mathbb{M}(a, Tr, t)]\mathbb{M}(a, Tr, t)}{(1 + 1)(\mathbb{M}(a, Tr, t) + 1)}, \\ \{1 \circledast \mathbb{M}(a, Tr, t) \circledast 1\} \end{Bmatrix}$$

$$\mathbb{M}(a, Tr, kt) \geq \mathbb{M}(a, Tr, t)$$

Using the same reason as above $a = Tr$

In the same way

$$\mathbb{N}(Sz, Tr, t) \leq max \begin{Bmatrix} \dfrac{2[\mathbb{N}(Qz, Sz, t) + \mathbb{N}(Pr, Tr, t)]\mathbb{N}(Pr, Sz, t)\mathbb{N}(Pr, Tr, t)}{(1 + \mathbb{N}(Qz, Sz, t))(\mathbb{N}(Pr, Tr, t) + 1)}, \\ \{\mathbb{N}(Qz, Sz, t) \boxdot \mathbb{N}(Pr, Tr, t) \boxdot \mathbb{N}(Pr, sz, t)\} \end{Bmatrix}$$

$$\mathbb{N}(a, Tr, t) \leq max \begin{Bmatrix} \dfrac{2[\mathbb{N}(a, a, t) + \mathbb{N}(a, Tr, t)]\mathbb{N}(a, a, t)\mathbb{N}(a, Tr, t)}{(1 + \mathbb{N}(a, a, t))(\mathbb{N}(a, Tr, t) + 1)}, \\ \{\mathbb{N}(Qz, Sz, t) \boxdot \mathbb{N}(Pr, Tr, t) \boxdot \mathbb{N}(Pr, sz, t)\} \end{Bmatrix}$$

$$\mathbb{N}(a, Tr, t) \leq max \begin{Bmatrix} \dfrac{2[\mathbb{N}(a, a, t) + \mathbb{N}(a, Tr, t)]\mathbb{N}(a, a, t)\mathbb{N}(a, Tr, t)}{(1 + \mathbb{N}(a, a, t))(\mathbb{N}(a, Tr, t) + 1)}, \\ \{\mathbb{N}(a, a, t) \boxdot \mathbb{N}(a, Tr, t) \boxdot \mathbb{N}(a, a, t)\} \end{Bmatrix},$$

$$\mathbb{N}(a, Tr, t) \leq max \begin{Bmatrix} 0, \\ \{0 \boxdot \mathbb{N}(a, Tr, t) \boxdot 0\} \end{Bmatrix}$$

Which implies that $a = Tr$

As we proved earlier $Sz = Qz = a$

It is given that Q, S is Q −intimate we have

$$\mathbb{M}(Qa, a, t) \leq \mathbb{M}(Sa, a, t), \mathbb{N}(Qa, a, t) \leq \mathbb{N}(Sa, a, t)$$

Let's consider $Sa \neq a$ & using conditions (a) & (b), using replacements $x = a, y = r$

$$\mathbb{M}(Sa, Tr, t) \geq min \begin{Bmatrix} \dfrac{2[\mathbb{M}(Qa, Sa, t) + \mathbb{M}(Pr, Tr, t)]\mathbb{M}(Pr, Sa, t)\mathbb{M}(Pr, Tr, t)}{(1 + \mathbb{M}(Qa, Sa, t))(\mathbb{M}(Pr, Tr, t) + 1)}, \\ \{\mathbb{M}(Qa, Sa, t) \circledast \mathbb{M}(Pr, Tr, t) \circledast \mathbb{M}(Pr, Sa, t)\} \end{Bmatrix}$$

$$\mathbb{M}(Sa,a,kt) \geq min\left\{\begin{matrix}\dfrac{2[\mathbb{M}(a,Sa,t)+\mathbb{M}(a,a,t)]\mathbb{M}(a,Sa,t)\mathbb{M}(a,a,t)}{(1+\mathbb{M}(Qa,Sa,t))(\mathbb{M}(Pr,Tr,t)+1)}, \\ \{\mathbb{M}(a,Sa,t) \circledast \mathbb{M}(a,a,t) \circledast \mathbb{M}(a,Sa,t)\}\end{matrix}\right\}$$

$$\mathbb{M}(Sa,a,kt) \geq min\left\{\begin{matrix}\dfrac{2[\mathbb{M}(a,Sa,t)+\mathbb{M}(a,a,t)]\mathbb{M}(a,Sa,t)\mathbb{M}(a,a,t)}{(1+\mathbb{M}(a,Sa,t))(\mathbb{M}(Pr,Tr,t)+1)}, \\ \{\mathbb{M}(a,Sa,t) \circledast \mathbb{M}(a,a,t) \circledast \mathbb{M}(a,Sa,t)\}\end{matrix}\right\}$$

$$\mathbb{M}(Sa,a,kt) \geq min\left\{\begin{matrix}\dfrac{2[\mathbb{M}(a,Sa,t)+1]\mathbb{M}(a,Sa,t)}{(1+\mathbb{M}(a,Sa,t))(\mathbb{M}(a,a,t)+1)}, \\ \{\mathbb{M}(a,Sa,t) \circledast \mathbb{M}(a,a,t) \circledast \mathbb{M}(a,Sa,t)\}\end{matrix}\right\}$$

$$\mathbb{M}(Sa,a,kt) \geq min\left\{\begin{matrix}\dfrac{2[\mathbb{M}(a,Sa,t)+1]\mathbb{M}(a,Sa,t)}{(1+\mathbb{M}(a,Sa,t))(\mathbb{M}(a,a,t)+1)}, \\ \{\mathbb{M}(a,Sa,t) \circledast \mathbb{M}(a,a,t) \circledast \mathbb{M}(a,Sa,t)\}\end{matrix}\right\}$$

$$\mathbb{M}(Sa,a,kt) \geq min\left\{\begin{matrix}\dfrac{2[\mathbb{M}(a,Sa,t)+1]\mathbb{M}(a,Sa,t)}{(1+\mathbb{M}(a,Sa,t))(\mathbb{M}(a,a,t)+1)}, \\ \{\mathbb{M}(a,Sa,t) \circledast 1 \circledast \mathbb{M}(a,Sa,t)\}\end{matrix}\right\}$$

$$\mathbb{M}(Sa,a,kt) \geq \mathbb{M}(Sa,a,t)$$

This implies that $Sa = a$

Consequently

$$\mathbb{N}(Sa,a,kt) \leq max\left\{\begin{matrix}\dfrac{2[\mathbb{N}(a,Sa,t)+\mathbb{N}(a,a,t)]\mathbb{N}(a,Sa,t)\mathbb{N}(a,a,t)}{(1+\mathbb{N}(Qa,Sa,t))(\mathbb{N}(Pr,Tr,t)+1)}, \\ \{\mathbb{N}(a,Sa,t) \boxdot \mathbb{N}(a,a,t) \boxdot \mathbb{N}(a,Sa,t)\}\end{matrix}\right\}$$

$$\mathbb{N}(Sa,a,kt) \leq max\left\{\begin{matrix}0, \\ \{\mathbb{N}(a,Sa,t) \boxdot 0 \boxdot \mathbb{N}(a,Sa,t)\}\end{matrix}\right\}$$

$$\mathbb{N}(Sa,a,kt) \leq \mathbb{N}(a,Sa,t)$$

Since $\gamma < 1$ Therefore $Sa = a = Qa$

In similar manner we can consider that $Ta \neq a$ and using conditions (a) and (b) using replacement $x = z$ and $y = a$

$$\mathbb{M}(Sz,Ta,kt) \leq min\left\{\begin{matrix}\dfrac{2[\mathbb{M}(Qa,Sa,t)+\mathbb{M}(Pa,Ta,t)]\mathbb{M}(Pa,Sz,t)\mathbb{M}(Pa,Ta,t)}{(1+\mathbb{M}(Qa,Sa,t))(\mathbb{M}(Pa,Ta,t)+1)}, \\ \{\mathbb{M}(Qa,Sa,t) \circledast \mathbb{M}(Pa,Ta,t) \circledast \mathbb{M}(pa,sz,t)\}\end{matrix}\right\}$$

$$\mathbb{M}(a,Ta,kt) \geq min\left\{\begin{matrix}\dfrac{2[\mathbb{M}(a,a,t)+\mathbb{M}(a,Ta,t)]\mathbb{M}(a,a,t)\mathbb{M}(a,Ta,t)}{(1+\mathbb{M}(a,a,t))(\mathbb{M}(a,Ta,t)+1)}, \\ \{\mathbb{M}(a,a,t) \circledast \mathbb{M}(a,Ta,t) \circledast \mathbb{M}(a,a,t)\}\end{matrix}\right\}$$

$$\mathbb{M}(a,Ta,kt) \geq min\left\{\begin{matrix}\dfrac{2[1+\mathbb{M}(a,Ta,t)]\mathbb{M}(a,Ta,t)}{(1+1)(\mathbb{M}(a,Ta,t)+1)}, \\ \{1 \circledast \mathbb{M}(a,Ta,t) \circledast 1\}\end{matrix}\right\}$$

$$\mathbb{M}(a,Ta,kt) \geq min\left\{\begin{matrix}1, \\ \{1 \circledast \mathbb{M}(a,Ta,t) \circledast 1\}\end{matrix}\right\}$$

$$\mathbb{M}(a,Ta,kt) \geq \mathbb{M}(a,Ta,t)$$

This implies that $a = Ta = Pa$

$$\mathbb{N}(Sz,Ta,kt) \leq max \left\{ \begin{matrix} \dfrac{2[\mathbb{N}(a,a,t) + \mathbb{N}(a,Ta,t)]\mathbb{N}(a,a,t)\mathbb{N}(a,Ta,t)}{(1+\mathbb{N}(a,a,t))(\mathbb{N}(a,Ta,t)+1)}, \\ \{\mathbb{N}(a,a,t) \boxdot \mathbb{N}(a,Ta,t) \boxdot \mathbb{N}(a,a,t)\} \end{matrix} \right\}$$

$$\mathbb{N}(a,Ta,kt) \leq max \left\{ \begin{matrix} 0, \\ \{0 \boxdot \mathbb{N}(a,Ta,t) \boxdot 0\} \end{matrix} \right\}$$

$$\mathbb{N}(a,Ta,kt) \geq \mathbb{N}(a,Ta,t)$$

This implies that $a = Ta = Pa$

Hence, we proved that $Ta = Pa = Sa = Qa = a$ it means a is a common fixed point of P, Q, S, T.

This completes the proof.

Uniqueness:

Let a and b are two common fixed points such that $a \neq b$ using equation (a) & (b)

$$\mathbb{M}(Sa,Tb,kt) = \mathbb{M}(Su,Tv,t) \geq min \left\{ \begin{matrix} \dfrac{2[\mathbb{M}(Qa,Sa,t) + \mathbb{M}(Pb,Tb,t)]\mathbb{M}(Pb,Sa,t)\mathbb{M}(Pb,Tb,t)}{(1+\mathbb{M}(Qa,Sa,t))(\mathbb{M}(Pb,Tb,t)+1)}, \\ \{\mathbb{M}(Qa,Sa,t) \circledast \mathbb{M}(Pb,Tb,t) \circledast \mathbb{M}(pb,sa,t)\} \end{matrix} \right\}$$

$$\mathbb{M}(a,b,kt) \geq min \left\{ \begin{matrix} \dfrac{2[\mathbb{M}(a,a,t) + \mathbb{M}(b,b,t)]\mathbb{M}(b,a,t)\mathbb{M}(b,b,t)}{(1+\mathbb{M}(a,a,t))(\mathbb{M}(b,b,t)+1)}, \\ \{\mathbb{M}(a,a,t) \circledast \mathbb{M}(b,b,t) \circledast \mathbb{M}(b,a,t)\} \end{matrix} \right\}$$

$$\mathbb{M}(a,b,kt) \geq min \left\{ \begin{matrix} \dfrac{2[1+1]\mathbb{M}(b,a,t)}{(1+1)(1+1)}, \\ \{1 \circledast 1 \circledast \mathbb{M}(b,a,t)\} \end{matrix} \right\}$$

$$\mathbb{M}(a,b,kt) \geq \mathbb{M}(b,a,t)$$

$$\mathbb{N}(Sa,Tb,kt) \leq max \left\{ \begin{matrix} \dfrac{2[\mathbb{N}(a,a,t) + \mathbb{N}(b,b,t)]\mathbb{N}(b,a,t)\mathbb{N}(b,b,t)}{(1+\mathbb{N}(a,a,t))(\mathbb{N}(b,b,t)+1)}, \\ \{\mathbb{N}(a,a,t) \circledast \mathbb{N}(b,b,t) \circledast \mathbb{N}(b,a,t)\} \end{matrix} \right\}$$

$$\mathbb{N}(a,b,kt) \leq max \left\{ \begin{matrix} 0, \\ \{0 \boxdot 0 \boxdot \mathbb{N}(b,a,t)\} \end{matrix} \right\}$$

$$\mathbb{N}(a,b,kt) \leq \mathbb{N}(b,a,t)$$

This implies that $a = b$ hence it proves uniqueness.

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