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RESULT WITH DISLOCATED 2-METRIC SPACE

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Abstract

The purpose of this paper is to present some fixed point theorem in dislocated quasi 2-metric space for expansive type mappings.

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Introduction and Preliminaries:

It is well known that Banach Contraction mappings principle is one of the pivotal results of analysis. Generalizations of this principle have been obtained in several directions .Dass and Gupta [1] generalized Banach's Contraction principle in metric space. Also Rhoades [2] established a partial ordering for various definitions of contractive mappings. In 2005, Zeyada Salunke [4] proved some results on fixed point in dislocated quasimetric spaces. In 2005, Zeyada et al.[3] established a fixed point theorem in dislocated quasimetric spaces. In 2008, Aage and Salunke [4] proved some results on fixed point in dislocated quasimetric spaces. Recently, Isufati [5], proved fixed point theorem for contractive type condition with rational expression in dislocated quasimetric spaces. The following definitions will be needed in the sequel.

Definition 1.1(See [3]). Let X be a nonempty set, and let $d : X \times X \to [0,\infty)$ be a function, called a distance function. One needs the following conditions:

(M1) $d(x,x) = 0$,

(M2) $d(x,y) = d(y,x) = 0$, then $x = y$

(M3) $d(x,y) = d(y,x)$,

(M4) $d(x,y) \le d(x,z) + d(z,y)$,

(M4)' $d(x,y) \le \max\{d(x,z), d(z,y)\}$, for all $x,y,z \in X$

.If d satisfies conditions (M1)-(M4), then it is called a metric on X . If d satisfies conditions (M1), (M2), and (M4), it is called a quasimetric on X . If it satisfies conditions (M2)-(M4) ((M2) and (M4)), it is called a dislocated metric (or simply d-metric) (a dislocated quasimetric (or simply dq-metric)) on X , respectively. If a metric d satisfies the strong triangle inequality (M)', then it is called an ultrametric.

Definition 1.2 (See [3]). A sequence $\{x_n\}_{n\in N}$ in dq-metric space (dislocated quasimetric space) (X,d) is called a Cauchy sequence if , for given $\varepsilon > 0$, there exists $n_0 \in N$ such that $d(x_m,x_n) < \varepsilon$ or $d(x_n,x_m) < \varepsilon$, that is , $\min\{d(x_m,x_n), d(x_n,x_m)\} < \varepsilon$ for all $m,n \ge n_0$.

Definition 1.3 (See [3]). A sequence $\{x_n\}_{n\in N}$ in dq-metric space [d-metric space] is said to be d-converge to $x \in X$ provided that

$$\lim_{n\to\infty} d(x_n,x) = \lim_{n\to\infty} d(x,x_n) = 0 \qquad (1.1)$$

In this case, x is called a dq-limit [d-limit] of $\{x_n\}$ and we write $x_n \to x$.

Definition 1.4 (See [3]). A dq-metric space (X,d) is called complete if every Cauchy sequence in it is a dq-convergent.

Main Results

In this paper, we prove some fixed point theorem for continuous mapping satisfying expansion condition in complete dq 2-metric space.This result is motivated by Saluja et.al [6]

Theorem 2.1: Let (X,d) be a complete dislocated 2-metric space and T a continuous mappings satisfying the following condition:

$$d(Tx,Ty,a) + \alpha\left[\frac{d(x,Ty,a)+d(y,Tx,a)}{1+d(x,Ty,a).d(y,Tx,a)}\right] \ge \beta\frac{d(x,Tx,a)[1+d(y,Ty,a)]}{1+d(x,y,a)} + \gamma d(x,y,a)$$

(2.1)

For all $x,y \in X$, $x \ne y$,where $\alpha,\beta,\gamma \ge 0$ are real constants and $\beta+\gamma > 1+2\alpha$, $\gamma > 1+\alpha$, a>0. Then T has a fixed point in X .

Proof: Choose $x_0 \in X$ be arbitrary, to define the iterative sequence $\{x_n\}_{n\in N}$ as follows and $Tx_n = x_{n-1}$ for $n = 1,2,3......$ Then, using (2.1) we obtain

$$d(Tx_{n+1},Tx_{n+2},a)+\alpha\left[\frac{d(x_{n+1},Tx_{n+2},a)+d(x_{n+2},Tx_{n+1},a)}{1+d(x_{n+1},Tx_{n+2},a).d(x_{n+2},Tx_{n+1},a)}\right]\geq\beta\frac{d(x_{n+1},Tx_{n+1},a)[1+d(x_{n+2},Tx_{n+2},a)]}{1+d(x_{n+1},x_{n+2},a)}$$
$$+\gamma d(x_{n+1},x_{n+2},a)$$
$$\Rightarrow d(x_n,x_{n+1},a)+\alpha\left[\frac{d(x_{n+1},x_{n+1},a)+d(x_{n+2},x_n,a)}{1+d(x_{n+1},x_{n+1},a).d(x_{n+2},x_n,a)}\right]\geq\beta\frac{d(x_{n+1},x_n,a)[1+d(x_{n+2},x_{n+1},a)]}{1+d(x_{n+1},x_{n+2},a)}+\gamma d(x_{n+1},x_{n+2},a)$$
$$\Rightarrow d(x_n,x_{n+1},a)+\alpha d(x_{n+2},x_n,a)\geq\beta d(x_n,x_{n+1},a)+\gamma d(x_{n+1},x_{n+2},a)$$
$$\Rightarrow d(x_n,x_{n+1},a)+\alpha d(x_n,x_{n+1},a)+\alpha d(x_{n+1},x_{n+2},a)\geq\beta d(x_n,x_{n+1},a)+\gamma d(x_{n+1},x_{n+2},a)$$
$$\Rightarrow(1+\alpha-\beta)d(x_n,x_{n+1},a)\geq(\gamma-\alpha)d(x_{n+1},x_{n+2},a)$$

The last inequality gives

$$d(x_{n+1},x_{n+2},a)\leq\left(\frac{1+\alpha-\beta}{\gamma-\alpha}\right)d(x_n,x_{n+1},a)$$
$$\leq kd(x_n,x_{n+1})$$
(2.2)

Where $k=\frac{(1+\alpha-\beta)}{(\gamma-\alpha)}<1$. Hence by induction, we obtain

$$d(x_{n+1},x_{n+2},a)\leq k^{n+1}d(x_0,x_1,a)$$

Note that, for $m,n\in N$ such that $m>n$ we have

$$d(x_m,x_n,a)\leq d(x_m,x_{m-1},a)+d(x_{m-1},x_{m-2},a)+.................+d(x_{n+1},x_n,a)$$
$$\leq\left[k^{m-1}+k^{m-2}+.....................................+k^n\right]d(x_0,x_1,a)$$
$$\leq k^n\left(1+k+k^2+......................+k^{m-n-1}\right)d(x_0,x_1,a)$$
$$\leq k^n\sum_{r=0}^{\infty}k^r d(x_0,x_1,a)$$
$$=\frac{k^n}{1-k}d(x_0,x_1,a) \qquad (2.3)$$

Since $0\leq k<1$, then as $n\to\infty$, $k^n(1-k)^{-1}\to 0$. Hence, $d(x_m,x_n)\to 0$ as $m,n\to\infty$. This forces that $\{x_n\}_{n\in N}$ is a Cauchy sequence in X. But X is a complete dislocated metric space; hence, $\{x_n\}_{n\in N}$ is d-converges. Call the d-limit $x^*\in X$. Then, $x_n\to x^*$ as $n\to\infty$. By continuity of T we have,

$$Tx^*=T\left(d-\lim_{n\to\infty}x_n\right)=d-\lim_{n\to\infty}Tx_n=d-\lim_{n\to\infty}x_{n-1}=x^*$$
(2.4)

That is, $Tx^*=x^*$; thus, T has a fixed point in X.

Uniqueness

Let y^* be another fixed point of T in X, then $Ty^*=y^*$ and $Tx^*=x^*$

$$d(Tx^*,Ty^*,a)+\alpha\left[\frac{d(x^*,Ty^*,a)+d(y^*,Tx^*,a)}{1+d(x^*,Ty^*,a).d(y^*,Tx^*,a)}\right]\geq\beta\frac{d(x^*,Tx^*,a)[1+d(y^*,Ty^*,a)]}{1+d(x^*,y^*,a)}+\gamma d(x^*,y^*,a)$$
(2.5)

This implies that

$$d(x^*,y^*,a)+\alpha\left[\frac{d(x^*,y^*,a)+d(y^*,x^*,a)}{1+d(x^*,y^*,a).d(y^*,x^*,a)}\right]\geq\beta\frac{d(x^*,x^*,a)[1+d(y^*,y^*,a)]}{1+d(x^*,y^*,a)}+\gamma d(x^*,y^*,a)$$
$$\Rightarrow d(x^*,y^*,a)+\frac{2\alpha d(x^*,y^*,a)}{1+\left[d(x^*,y^*,a)\right]^2}\geq\gamma d(x^*,y^*,a)$$
$$\Rightarrow d(x^*,y^*,a)+\left[d(x^*,y^*,a)\right]^3+2\alpha d(x^*,y^*,a)\geq\gamma d(x^*,y^*,a)+\gamma\left[d(x^*,y^*,a)\right]^3$$
$$\Rightarrow(1+2\alpha-\gamma)d(x^*,y^*,a)\geq(\gamma-1)\left[d(x^*,y^*,a)\right]^3$$
$$d(x^*,y^*,a)\leq\left(\frac{1+2\alpha-\gamma}{\gamma-1}\right)^{\frac{1}{3}}d(x^*,y^*,a)$$
(2.6)

This is true only when $d(x^*,x^*,a)=0$. Similarly $d(y^*,x^*,a)=0$ .Hence $d(x^*,y^*,a)=d(y^*,x^*,a)=0$ and so $x^*=y^*$. Hence, T has a unique fixed point in X .

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