

AN ANALYTICAL RESEARCH IN HIGH FREQUENCY DIFFRACTION BY PERFECTLY CONDUCTING WEDGE

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Abstract

This Paper deals with high frequency diffraction of wave by a perfectly conducting wedge. The canonical problem of high frequency pulsed field diffraction has been analyzed via local scattering models, where a collimated wave packet (pulsed beam, PB) impinges on a perfectly conducting wedge. The space time localization is used and approximate expressions are derived. The space time forms depend explicitly on the local properties of the incident wave packet. The model is also extended to accommodate the diffraction of astigmatic wave packets. They parameterize the scattering mechanism in terms of tractable phenomenon. The presented diffraction problem, however exhibits richer scattering phenomenon and greater resolution in space-time.

Keywords-High Frequency, Diffraction, Pulsed beam, Wedge Propagation, Scattering Mechanism.

Introduction

The treatment of scattering characteristics from electromagnetic (EM) wave object interactions using modal Solutions are limited to structures whose surfaces can be described by orthogonal curvilinear coordinates. Moreover, most of the solutions are in the form of infinite series, which are poorly convergent when the dimensions of the object are greater than a few wavelengths. These limitations, therefore, exclude closed-form analyses of many practical scattering systems. When the dimensions of the scattering object are many wavelengths, high frequency asymptotic (HFA) techniques can be used to analyze many problems that are otherwise mathematically intractable. Two such techniques, which have received considerable attention in the past five decades, are the ray-type geometrical theory of diffraction (GTD) and the wave- (i.e., induced-source)-based physical theory of diffraction (PTD). Usually the diffraction phenomenon is understood as a deviation in a wave behavior from the geometrical optics (GO) laws. Just in this sense this term was introduced by Grimaldi who was the first to observe and describe the diffraction of light almost three hundred fifty years ago . Since then this phenomenon has been continuously investigated. An extensive study of HFA diffraction was stimulated by invention and development of

radar in the 1940s. Milestone events happened in 1957 and 1962 with

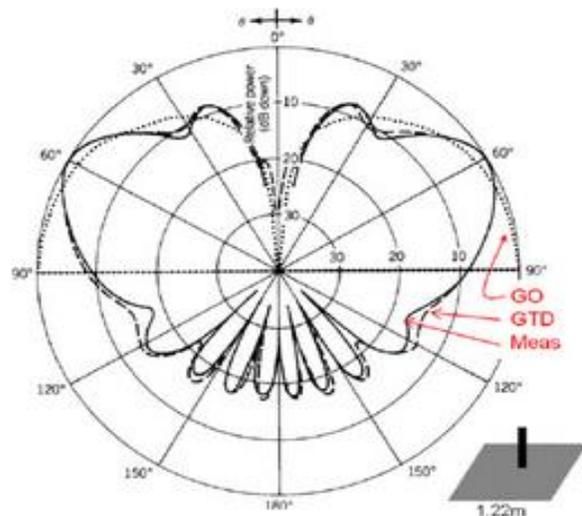


Fig- Radiation pattern of a vertical monopole above a PEC ground ($f/1\text{GHz}$)

Publications of papers on GTD and PTD. One may believe that these techniques are different and independent. However, a basic connection exists between them; they have a common foundation. They are based on the localization principle that in the high frequency limit, the diffracted field in vicinity of a scattering object depends on its local properties. However, this principle is utilized in PTD and GTD in different ways. In PTD, it is applied to the field (currents) on the object surface while in GTD to the diffracted rays, i.e. only to the ray-part of the field radiated by these currents. Thus, already on this fundamental level it is understood that GTD can be interpreted as the asymptotic ray form of PTD. That is why it is not surprising that the GTD edge diffracted rays are derived from the PTD field integrals by asymptotic evaluation.

Geometrical/Uniform Theory of Diffraction

The GTD, originated by Keller, and extended to UTD by Kouyoumjian and Pathak to remove the discontinuities along the incident and reflection shadow boundaries (ISB, RSB), is an extension of the classical GO, and it overcomes some of the limitations of GO by introducing a diffraction mechanism. At high frequencies, diffraction like reflection and refraction is a local phenomenon and it depends on the geometry of the object at the point of diffraction (e.g., edge, vertex, curved surface) and the amplitude, phase, and polarization of the incident field at the point of Diffraction. A field is associated with each diffracted ray, and the total field at a point is the sum of all the rays at that point. Some of the diffracted rays enter the shadow regions and account for the field intensity there. The diffracted field,

Which is determined by a generalization of Fermat's principle, is initiated at points on the surface of the object that create a discontinuity in the incident GO field (ISB and RSB). The phase of the field on a ray is assumed to be equal to the product of the optical/geometrical length of the ray from some reference point and the wave number of the medium. Appropriate phase jumps must be added as rays pass through caustics. The amplitude is assumed to vary in accordance with the principle of conservation of energy in a narrow tube of rays. The initial value of the field on a diffracted ray is determined from the incident field with the aid of an appropriate diffraction coefficient that is a dyadic for EM fields. This is analogous to the manner reflected fields are determined using the reflection coefficient. The rays also

follow paths that make the optical distance from the source to the observation point an extremum (usually a minimum). This leads to straight-line propagation within homogeneous media and along geodesics (surface extrema) on smooth surfaces. The field intensity also attenuates exponentially as it travels along surface geodesics. The diffraction and attenuation coefficients are usually determined from the asymptotic solutions of the simplest boundary-value problems, which have the same local geometry at the points of diffraction on the object at the points of interest. Geometries of this type are referred to as canonical problems. One of the simplest geometries that will be discussed in this paper is a conducting wedge.

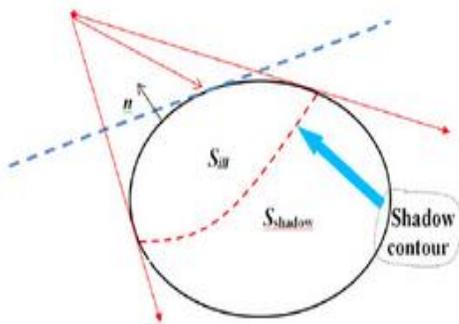


Fig 1-EM field-object interaction

The primary objective in using GTD is to resolve each problem to smaller components, each representing a canonical geometry with a known solution. The ultimate solution is a superposition of the contributions from each canonical problem. The contribution of diffraction is illustrated with the example presented in Figure. Here, a quarter wavelength, bottom-fed vertical monopole antenna is erected above a finite size square perfectly electrical conductor (PEC) ground plane. The finite ground plane introduces not only reflections but also significant diffractions as shown in the figure.

Physical Theory of Diffraction

PTD is extension of the physical optics (PO) approximation introduced by Kirchoff for scalar waves and by Macdonald for EM waves. Here, we expose PO and PTD for scalar/acoustic waves and utilize acoustic terminology. A scattering object is considered perfectly reflecting with soft (Dirichlet) or hard (Neumann) boundary conditions. According to PO, the total field on the illuminated side of scattering object is assumed to be the same as on the infinite tangential plane. The object is convex; its radii of curvature are large compared to the wavelength. On the shadow side, the total field is zero. This approximation is a powerful tool widely used in acoustics, optics, and EMs. In particular, it is extensively applied in the design of reflector antennas and in calculations of scattering from objects. The PO approximation describes properly both all reflected rays away from the GO boundaries and the diffracted field near these boundaries, as well as near foci and caustics. The PO drawbacks are the following. First, it is not self-consistent: When the observation point approaches the scattering surface, the PO integrals do not reproduce the initial values for the surface field. Second, the PO field does not satisfy rigorously the boundary conditions and the reciprocity principle.

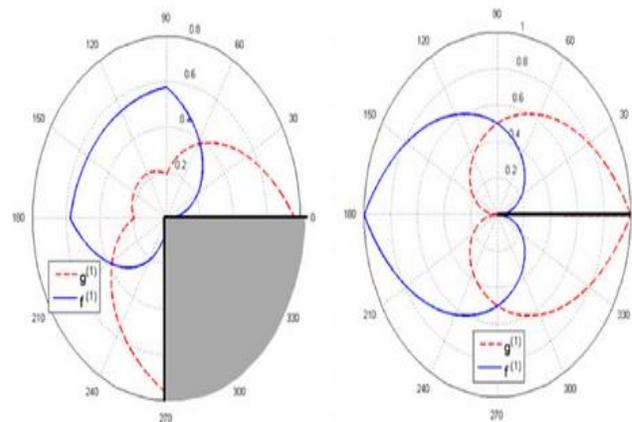


Fig 2- Directivity patterns of f^1, g^1

The reason for these shortcomings is the GO approximation for the surface field, which does not include its diffraction components. The PO shortcomings are overcome by PTD, which improves PO by taking into account the diffracted surface field.

Test Problems and GTD/UTD-PTD Comparisons

The GTD/UTD and PTD models are compared on two canonical diffraction problems.

A) Wedge Scattering Problem

The wedge scattering problem presents a canonical problem where all HFA techniques can be investigated and comparisons can be performed. A series of papers published recently both review all the HFA techniques and present an attractive virtual diffraction tool that can be used for teaching/training as well as research.

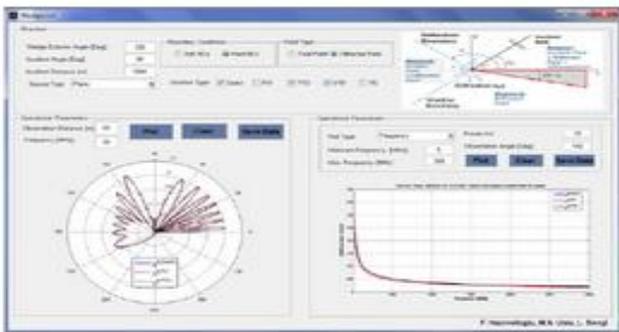


Fig-3 WedgeGUI diffraction tool presented

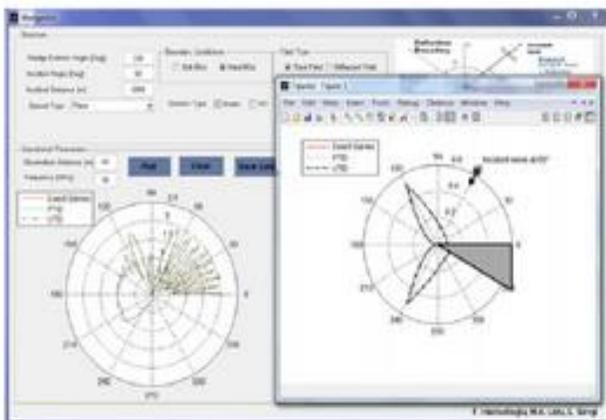


Fig-4 Total and diffracted fields around the wedge calculated via exact mode summation, UTD and PTD methods

Fig 3 and 4 belong to this virtual tool. One of the main interests of diffraction by wedges is that engineers and scientists have investigated how the shape and material properties of complex structures affect the backscattering characteristics.

The attraction in this area is primarily aimed toward designs of low-profile (stealth) technology by using appropriate shaping along with lossy or coated materials to reduce the radar visibility, as represented by radar cross-section (RCS), of complex radar targets, such as aircraft, spacecraft, and missiles.

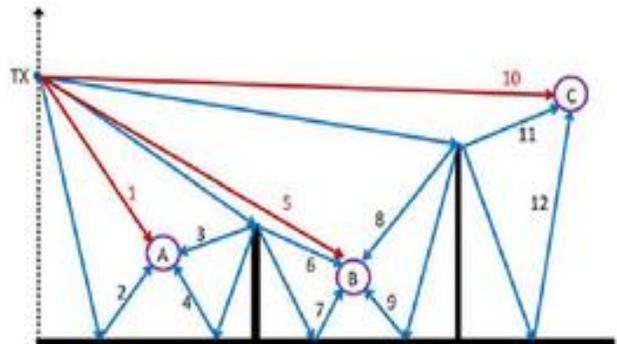


Fig-5 Flat-earth and double knife-edge problem

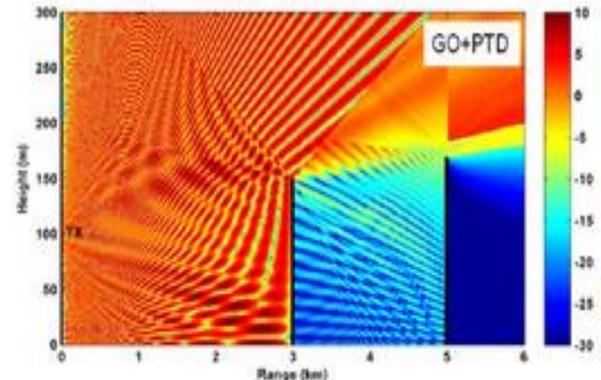
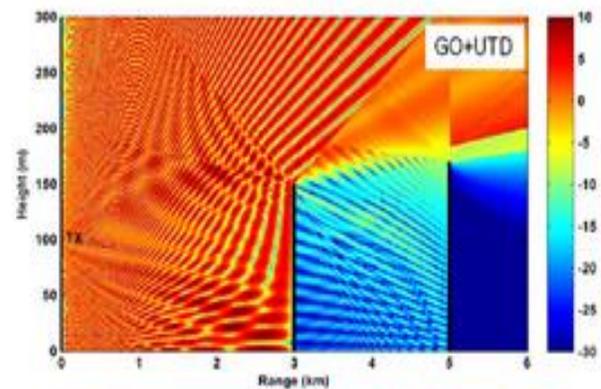


Fig-6 3D field maps for edges at 3 and 5 km ranges

B) Flat-Earth and Double Knife-Edge Problem

The scenario of this problem is shown Fig 5 showing possible rays at different observation points (A, B, and C). This is a complicated problem which includes extraction of GO eigenrays (source-emanating rays which reach/pass the receiver) plus calculation of reflected and diffracted fields for the specified source and observer locations. Typical simulations are performed with GO plus both GTD/UTD and PTD with 150 m- and 175 m-high double knife-edges at 3 km and 5 km ranges, respectively. The line source is 100 m above the ground with HBC. The frequency is 300 MHz. Results are illustrated in Fig 6 and 7.

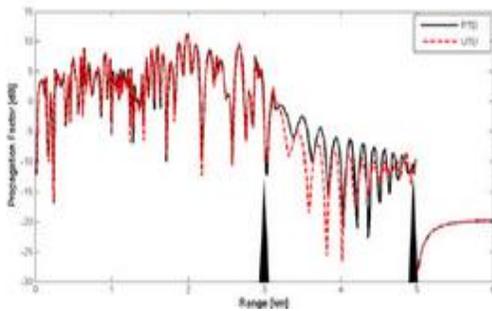


Fig-7 Propagation factor vs. range

Perfectly Conducting Wedge

A) 2-D perfectly conducting wedge

In this paper, we use a completely different approach to treat the problem of 2-D electrostatic image problems for the perfectly conducting wedge with arbitrary angles. We will show that for this problem, using the concept of “fractional-order” multipoles, it is possible to describe “images” that effectively behave as distributed “intermediate” cases between those discrete images obtained for specific wedge angles. The fractional orders of these equivalent “images,” which we call “fractional” images, depend on the wedge’s angle. We also extend this image method for the 3-D perfectly conducting cones.

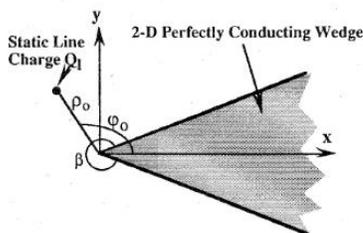


Fig-8 Two-dimensional perfectly conducting wedge with the outer angle

Consider a 2-D perfectly conducting wedge with the outer angle of β . A Cartesian coordinate system (x, y, z) is chosen with the x axis along the edge of the wedge, and the $x-z$ plane being the symmetry plane of the wedge. (Fig 8). A cylindrical coordinate system (ρ, γ, z) is also used with $x = \rho \cos \gamma$ and $y = \rho \sin \gamma$. An infinitely long uniform static line charge with charge density per unit length of Q_l (Coulomb/m) is located parallel with the z axis at an arbitrary point with coordinates (P_0, γ_0) . The electrostatic potential $\Phi_{lw}(\rho, \gamma)$ due to this line charge in front of the perfectly conducting wedge can be written as

$$\Phi_{lw}(\rho, \varphi) = \sum_{m=1}^{\infty} \frac{Q_l}{m\pi\epsilon} \left(\frac{\rho <}{\rho >} \right)^{m\pi/\beta} \cdot \sin \left[\frac{m\pi}{2} - \frac{m\pi}{\beta} (\pi - \varphi) \right] \cdot \sin \left[\frac{m\pi}{2} - \frac{m\pi}{\beta} (\pi - \varphi_0) \right]$$

Acknowledgments

In this paper, we have introduced and described, in detail, the concept of “fractional” image methods for the electrostatic problem involving the perfectly conducting wedge. Using fractional calculus and the concept of “fractional-order” poles, we have shown that the electrostatic potentials in front of perfectly conducting wedges can be expressed equivalently as the potentials of sets of equivalent charges that have the form of fractional-order poles. The “order” of these poles depends on the wedge angle (for the 2-D wedge problem). We have also shown that the potential of these general “fractional images” approach that of the well known discrete images for the special cases.

These fractional “image” charges may have applications in the analysis of electrostatic problems involving these structures. For example, one may speculate that in numerical evaluation of fields in the presence of sharp edges and tips, the knowledge of the form of these equivalent “image” charges may become useful. Extension of this method to the electrostatic 2-D dielectric wedge is currently under study by the author.



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