FIXED POINT RESULTS ON COMPLETE RANDOM G-METRIC SPACE FOR INTEGRAL TYPE MAPPINGS

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Abstract

In the present paper some fixed point theorems are established in random G-Metric space for integral type mappings. The results are obtained from the basic definitions of G- metric space and extended for integral type mappings

Keywords: Fixed point, Random G metric space
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Introduction and preliminaries

The first well known result on fixed point for contraction mapping was Banach fixed point hypothesis , Published in 1922 .In general the theory of Complete G-Metric Space ,the author Ghaler and Dhage proves the hypotheses on contraction mappings existing the subsequent results.

Many different generalizations can be done as past years as a development of invariant point theory in Metric space. The researchers Gahler and Dhage describes the notion of metric space and generalized many results related to the spaces 2-metric and D-metric.The Metric space conception is generalized in different ways has been proposed by many authors by Gahler and Dhage they obtain the results for 2-Metric space are independent and also proves the generalization and corresponding many results in metric spaces.The author Dhage develops the theory of D-Metric space and generalized many results for single and multivalued mappings of ordinary metric functions. Authors Abbas and Rhoades introduced the concept of communal fixed point theory in general metric space with distinct mappings gratifying the property P in G-Metric space. Also the author Mustafa.et. al. presents some theorems offixed point for mapping with different contractive condition.

In recent years, the study of fixed points of random operators forms a central topic in probabilistic functional analysis. In 1950 study of probabilistic was initiated in The Prague School of probabilistic. After the publication of the survey articles of Bharucha-Reid so many research works have been started in this area. Then many interesting random fixed point results and applications have appeared in the literature- for example the work of Beg and Shahazad, Lin (particularly random iteration schemes leading to random fixed point of random operators have been elaborately discussed in B.S. Choudhary, and M. Ray, B.S. Choudhary, and A. Upadhyay, V.B. Dhagat , A.Sharma, & R.K Bhardwaj and others.
In the present paper using above concepts a new concept of random G metric spaces is stablished for integral type mappings, which is the modification of G metric space using random operator. Now the modifications of the definitions are as folows:

Definition 2.1: Suppose U be a non-empty set, let a mapping$G : U \times U \times U \to R^+$ be a functional which satisfy the subsequent axioms:

1] G_1 $G(\xi u, \xi v, \xi w) = 0$ $If \xi u = \xi v = \xi w.$

2] G_2 $0 < G(\xi u, \xi v,\ \xi w)$ $\forall\, \xi\, u, \xi v \in U$ $with$ $\xi u \neq \xi v.$

3] G_3 $G(\xi u, \xi v, \xi\, w) \leq$ $G(\xi u, \xi v, \xi\, w),$ $\forall$ $\xi u, \xi v, \xi w \in U$ $with\, \xi\, w \neq \xi v$

4] G_4 $G(\xi u, \xi v,\ \xi w) =$ $G(\xi u, \xi w, \xi\, v) = G(\xi v,\ \xi w,\ \xi u)$

(Symmetry in all three variables)

5] G_5 $G(\xi u, \xi v, \xi w) \leq G(\xi u, \xi x, \xi x) + G(\xi x, \xi v, \xi w), \ \forall \ \xi u, \xi v, \xi w \in U$

(Rectangular inequality)

Therefore the functional G is known as random G-Metric, with the couple (U, G).

Definition 2.2: Suppose a metric space (U, G) be random G-metric then a classification$\{\xi x_n\}$ is named as random G- Cauchy, for every $\in > 0, \exists \ n \in N$ such that $G(\xi x_n, \xi x_m, \xi x_l) < \in, \ \ \forall \ n, \ m, l \in N$

i.e. $G(\xi x_n, \xi x_m, \xi x_l) \to 0 \quad as \ n, m, l \to \infty$

Proposition 2.3: Suppose a metric space (U, G) be G-metric, also a sequence$\{x_n\}$ be a point of U, and then the sequence$\{x_n\}$ is G-Convergent to x, If

$\lim_{n,m\to\infty} G(x, x_n, x_m) = 0 \quad \forall \in > 0 \ \exists \ n \in N$ Such that

$$G(x, x_n, x_m) < \in \qquad \forall \ n, m \geq M$$

Proposition 2.4: Suppose a metric space (U, G) be random G-metric, therefore the subsequent results are Comparable as

(i) Classification $\{\xi x_n\}$ is G- Converges to ξx.
(ii) $G(\xi x_n, \xi x_n, \xi x) \to 0 \ as \ n \to \infty$
(iii) $G(\xi x_n, \xi x, \xi x) \to 0 \ as \ n \to \infty$

Proposition 2.5: Suppose a metric space (U, G) and (U',G') be two Random G-metrics then S is a aligning from X to X' is at a point which is G- unceasing, and also it is unceasing at $\xi \ x$ If a metric space is raomdom G- Sequentially, wherever $\{\xi x_n\}$ is G- convergent to ξx, $\{f(\xi x_n)\}$is G- converges to $f(\xi x)$.

Definition2.6: Suppose the spaces (U, G) and (U', G') be double random G-metric and let S :(U, G) → (U',G') be a functional , then the function f is known to be random G- unceasing at a point $\xi a \in U$, Uncertainty assumed $\in > 0, \exists \ \delta > 0$ such that $\xi x, \xi y \in U$; $G(\xi a, \xi x, \xi y) < \delta$

This implies that $G\big(f(\xi a), f(\xi x), f(\xi y)\big) < \in$, A functional f is G- unceasing on X iff it is G- unceasing at all $\xi a \in U$.

Definition 2.7: Suppose a metric space (U, G) be random G-metric is known to be random G- complete, If for all random G- Cauchy classification in (U, G) is random G- converges in (U, G).

Throughout this paper, (Ω, Σ) denotes a measurable space consisting of a set Ω and sigma algebra Σ of subset of Ω . U stands for a complete metric space, C is nonempty subset of U

Definition 2.8: A function R : $\Omega \times C \to C$ is said to be measurable if

$R^{-1}(B \cap C) \in \Sigma$ for every Borel subset B of U

Definition 2.9: A function R : $\Omega \times C \to C$ is said to be random operator, if

R(., x) : $\Omega \to C$ is measurable for every $x \in C$.

Definition 2.10: A random operator R : $\Omega \times C \to C$ is said to be continuous if for fixed $\xi \in \Omega, R(\xi, .) : C \to C$ is continuous.

Definition 2.11 :. A measurable function $g : \Omega \to C$ is said to be random fixed point of the random operator R : $\Omega \times C \to C$, if $R\big(\xi, g(\xi)\big) = g(\xi)$, □$\xi \in \Omega$□□or R(ξx) = ξx

Theorem 2.12 (Branciari) (Branciari, A. 2002) **:** In 2002, A. Branciari (Branciari, A., 2002) analyzed the existence of fixed point for mapping f defined on a complete metric space (X, d) satisfying a general contractive condition of integral type.
Let (X, d) be a complete metric space, $c \in (0,1)$ and let $R : X \to X$ be a mapping such that for each $x, y \in X$,

$$\int_0^{d(Rx,Ry)} \varphi(t)\, dt \le c \int_0^{d(x,y)} \varphi(t)\, dt \qquad \text{(2.12(i))}$$

Where $\varphi: [0, +\infty) \to [0, +\infty)$ is a Lesbesgue-integrable mapping which is summable on each compact subset of *[0, +∞)*, nonnegative, and such that for each $\epsilon > 0$, $\int_0^{\epsilon} \varphi(t) > 0$, then R has a unique fixed point $a \in X$ such that for each

$$x \in X,\ lim_{n\to\infty} R^n x = a$$

Proposition 2.13: Suppose a metric space (U, G) be random G-metric, therefore the functional $G(\xi x, \xi y, \xi z)$ is jointly continuous in all three of its variable.

Theorem 2.14 : Suppose a metric space (U, G) be G-metric which is complete, let $\alpha \in [0,1)$ and $G : U \to U$ be a map then we have for each $x, y \in X$

$$d(Gx, Gy) \le \alpha\, d(x, y)$$

Then, T has a unique invariant point $z \in U$ such that for each $x \in U, \lim_{n\to\infty} G^n x = z.$

After this result, more theorems with contraction mapping proved different forms of contractive inequalities.

Theorem 2.15: Suppose a metric space (U, G) be G-metric which is complete, let $\alpha \in [0,1)$ and $G : U \to U$ be a map then we have for each $c \in U$,

$$\int_0^{d(G\xi x,\ G\xi y)} \psi(t)dt \le \int_0^{d(\xi x, \xi y)} \psi(t)dt$$

Where $\psi: [0, +\infty] \to [0, +\infty]$ is a aligning which is the sum on each compact subclass of $[0, +\infty]$, is lebesgue integrable and Non-destructive, such that

$$\forall\ \varepsilon > 0, \qquad \int_0^{\varepsilon} \psi(t)dt$$

Then, T has Unique random fixed point $\xi z \in G$, such that for each $\xi x \in U$, $G^n \xi x = \xi z$ as $n \to \infty$.

Main Results

Theorem 3.1: Suppose a metric (U, G) space be G- Complete or it is Complete G-Metric then for a mapping $S : U \to U$ which gratifies the subsequent condition for all $x, y, z \in U$.

$$\int_0^{G(S(\xi x), S(\xi y), S(\xi z))} \psi(t)dt$$

$$\le q\, max \int_0^{\left\{\begin{matrix} G(\xi x,\xi y,\xi z), G(\xi x, S(\xi x), S(\xi y)), G(\xi y, S(\xi y), S(\xi z)), \\ G(\xi z, S(\xi z), S(\xi x)), G(\xi x, S(\xi y), S(\xi z)), G(\xi y, S(\xi x), S(\xi z)) \\ G(\xi z, S(\xi x), S(\xi y)), G(\xi x, S(\xi x), S(\xi x)), G(\xi y, S(\xi y), S(\xi y)), \\ G(\xi z, S(\xi z), S(\xi z)), G(\xi x, S(\xi y), S(\xi y)), G(\xi y, S\xi(z), S(\xi z)), \\ G(\xi z, S(\xi x), S(\xi x)), G(\xi x, \xi y, S(\xi z)), \\ G(\xi x, \xi z, S(\xi y)), G(\xi z, \xi y, S(\xi x)) \end{matrix}\right\}} \psi(t)dt$$

[3.1 (i)]

Where $q > 0, 0 < q < 1$ *and* $\psi : (0,1) \to (0,1), (0,1]$ is a aligning which is the sum on each compact subclass of $[0, +\infty]$, is lebesgue integrable and Non-destructive, such that

$\forall\ \varepsilon > 0$,

$$\int_0^{\epsilon} \psi(t)dt > 0\ , \forall \epsilon > 0 \qquad [[3.1\ (ii)]$$

Where $q \in \left(0, {}^{1}/_{2}\right]$ then S has a unique fixed point such that for each $x \in U$, $S^n x = z$ as $n \to \infty$ and S is G- continuous at u.

Proof: Let S satisfies Eq. [3.1 (i)] let x_0 be any random point in U, then we express a sequence $\{x_n\}$ in U as

$$\xi x_n = S^n(\xi x_0)$$

Then from [3.1 (i)] we have

$$\int_0^{G(\xi x_n, \xi x_{n+1}, \xi x_{n+1})} \psi(t)dt$$

$$\leq q \max \int_0^{\left\{\begin{matrix} G(\xi x_{n-1}\xi, x_n, \xi x_n), G(\xi x_{n-1}, \xi x_n, \xi x_{n+1}), G(\xi x_n, \xi x_{n+1}, \xi x_{n+1}), \\ G(\xi x_n, \xi x_{n+1}, \xi x_n), G(\xi x_{n-1}, \xi x_{n+1}, \xi x_{n+1}), G(\xi x_n, \xi x_{n+1}, \xi x_{n+1}) \\ G(\xi x_n, \xi x_n, \xi x_{n+1}), G(\xi x_{n-1}, \xi x_n, \xi x_n), G(\xi x_n, \xi x_{n+1}, \xi x_{n+1}), \\ G(\xi x_n, \xi x_{n+1}, \xi x_{n+1}), G(\xi x_{n-1}, \xi\, x_{n+1}, \xi x_{n+1}), G(\xi x_n, \xi x_{n+1}, \xi x_{n+1}), \\ G(\xi x_n, \xi x_n, \xi x_n), G(\xi x_{n-1}, \xi x_n, \xi x_{n+1}), G(\xi x_{n-1}, \xi x_n, \xi x_{n+1}), \\ G(\xi x_n, \xi x_n \xi, x_{n-1}) \end{matrix}\right\}} \psi(t)dt$$

$$\leq q \max \int_0^{\left\{\begin{matrix} G(\xi x_{n-1}, \xi x_n, \xi x_n), G(\xi x_n, \xi x_{n+1}, \xi x_{n+1}), G(\xi x_{n-1}, \xi x_{n+1}, \xi x_{n+1}), \\ G(\xi x_{n-1}, \xi x_n, \xi x_{n+1}), G(\xi x_{n+1}, \xi x_n, \xi x_n) \end{matrix}\right\}} \psi(t)dt$$

[3.1 (iii)]

So,

$$\int_0^{G(\xi x_n, \xi x_{n+1}, \xi x_{n+1})} \psi(t)dt \leq q \max \int_0^{\left\{\begin{matrix} G(\xi x_{n-1}, \xi x_n, \xi x_n), G(\xi x_{n-1}, \xi x_{n+1}, \xi x_{n+1}), \\ G(\xi x_{n+1}, \xi x_n, \xi x_n), G(\xi x_{n-1}, \xi x_n, \xi x_{n+1}) \end{matrix}\right\}} \psi(t)dt$$

[3.1 (iv)]

Since, By G(5) we have

$$\int_0^{G(\xi x_{n-1}, \xi\, x_n, \xi x_n)} \psi(t)dt \leq q \max \int_0^{[G(\xi x_{n-1}, \xi\, x_n, \xi x_n) + G(\xi x_n, \xi x_{n-1}, \xi x_{n+1})]} \psi(t)dt$$

[3.1 (v)]

Also,

$$\int_0^{G(\xi x_{n-1}, \xi x_n, \xi x_{n+1})} \psi(t)dt \leq q \max \int_0^{[G(\xi x_{n-1}, \xi x_n, \xi x_n) + G(\xi x_n, \xi x_n, \xi x_{n+1})]} \psi(t)dt$$

That implies

$$\int_0^{G(\xi x_n, \xi x_{n+1}, \xi x_{n+1})} \psi(t)dt$$

$$\leq q \max \int_0^{\left\{\begin{matrix} G(\xi x_{n-1}, \xi x_n, \xi x_n), G(\xi x_n, \xi x_n, \xi x_{n+1}) + G(\xi x_n, \xi x_{n+1}, \xi x_{n+1}), \\ G(\xi x_{n+1}, \xi x_n, \xi x_n), G(\xi x_{n-1}, \xi x_n, \xi x_n) + G(\xi x_n, \xi x_n, \xi x_{n+1}) \end{matrix}\right\}} \psi(t)dt$$

$$\int_0^{G(\xi x_n, \xi x_{n+1}, \xi x_{n+1})} \psi(t)dt$$

$$\leq q \max \int_0^{\left\{\begin{matrix} G(\xi x_{n-1}, \xi x_n, \xi x_n), G(\xi x_n, \xi x_n, \xi x_{n+1}) + G(\xi x_n, \xi x_{n+1}, \xi x_{n+1}), \\ G(\xi x_{n+1}, \xi x_n, \xi x_n), G(\xi x_{n-1}, \xi x_n, \xi x_n) + G(\xi x_n, \xi x_n, \xi x_{n+1}) \end{matrix}\right\}} \psi(t)dt$$

$$\leq q\ max \int_0^{\left\{\begin{matrix} G(\xi x_{n-1},\xi x_n,\xi x_n)+G(\xi x_n,\xi x_{n+1},\xi x_{n+1}), \\ G(\xi x_{n-1},\xi x_n,x_n)+G(\xi x_n,\xi x_n,\xi x_{n+1}) \end{matrix}\right\}} \psi(t)dt$$

By G(3), $\int_0^{G(\xi x_n,\xi x_{n+1}\xi,x_{n+1})} \psi(t)dt \leq \int_0^{G(\xi x_n,\xi x_{n+1}\xi,x_{n+1})} \psi(t)dt$

$$\int_0^{G(\xi x_n,\xi x_{n+1},\xi x_{n+1})} \psi(t)dt \leq q\ max \int_0^{\left\{\begin{matrix} G(\xi x_{n-1},\xi x_n,\xi x_n)+G(\xi x_n,\xi x_{n+1},\xi x_{n+1}), \\ G(\xi x_{n-1},\xi x_n,\xi x_n)+G(\xi x_n,\xi x_{n+1},\xi x_{n+1}) \end{matrix}\right\}} \psi(t)dt$$

[3.1 (vi)]

Therefore

$$\int_0^{G(\xi x_n,\xi x_{n+1},\xi x_{n+1})} \psi(t)dt \leq q \int_0^{[G\xi(x_{n-1}\xi,x_n,\xi x_n)+G(\xi x_n,\xi x_{n+1},\xi x_{n+1})]} \psi(t)dt$$

[3.1 (vii)]

$$\int_0^{G(\xi x_n,\xi x_{n+1},\xi x_{n+1})} \psi(t)dt \leq \frac{q}{1-q} \int_0^{[G(\xi x_{n-1},\xi x_n,\xi x_n)]} \psi(t)dt$$

[3.1 (viii)]

Let $k = \frac{q}{1-q}$, then for $k < 1$ and by repeated process of the application of Eq. [8.2.3.6], we have

$$\int_0^{G(\xi x_n,\xi x_{n+1},\xi x_{n+1})} \psi(t)dt \leq k^n \int_0^{G(\xi x_0,\xi x_1,\xi x_1)} \psi(t)dt$$

[3.1 (ix)]

Then for all $n, m \in N, n \leq m$, we have by again use of four-sided inequality and Eq[3.1 (ix)]
We have

$$G(\xi x_n, \xi x_m, \xi x_m) \leq G(\xi x_n, \xi x_{n+1}, \xi x_{n+1}) + G(\xi x_{n+1}, \xi x_{n+2}, \xi x_{n+2}) + G(\xi x_{n+2}, \xi x_{n+3}, \xi x_{n+3}) + \dots \dots \dots \dots \dots \dots \dots \dots \dots + G(\xi x_{m-1}, \xi x_m, \xi x_m)$$

$$\leq (k^n + k^{n+1} + \cdots \dots \dots \dots \dots + k^{m-1})\ G(\xi x_0, \xi x_1, \xi x_1)$$

[3.1 (x)]

$$\leq \frac{k^n}{1-k}\ G(\xi x_0, \xi x_1, \xi x_1)$$

Then, $\lim_{n,m\to\infty} G(\xi x_n, \xi x_m, \xi x_m) = 0$

Since $\lim_{n\to\infty} \frac{k^n}{1-k}\ G(\xi x_0, \xi x_1, \xi x_1) = 0$ for all $n, m \in N$
Therefore by prop (G 5) we assume that

$$\int_0^{G(\xi x_n,\xi\, x_m,\xi x_l)} \psi(t)dt \leq \int_0^{G(\xi x_n,\xi\, x_m,\xi x_m)+G(\xi x_l\xi,x_m\xi,x_m)} \psi(t)dt$$

Now at limit $n, m, l \to \infty$ we get

$$G(\xi x_n, \xi x_m, \xi x_l) \to 0$$

So, the classification $\{\xi x_n\}$ is a random G- Cauchy classification, , by extensiveness of random G- metric space (U, G), there exist $\xi v \in U$ such that the classification $\{\xi x_n\}$ is G-converges to ξv.
Let $S(\xi v) \neq \xi v$, then

$$\int_0^{G(\xi x_n,S(\xi v),S(\xi v))} \psi(t)dt$$

$$\leq q\max\int_0^{\left\{\begin{array}{c}G(\xi x_{n-1},\xi v,\xi v),G(\xi x_{n-1},\xi x_n,S(\xi v)),G(\xi v,S(\xi v),S(\xi v)),\\ G(\xi v,S(\xi v),\xi x_n),G(\xi x_{n-1},S\xi(v),S(\xi v)),G(\xi v,\xi x_n,S(\xi v))\\ G(\xi v,\xi x_n,S(\xi v)),G(\xi x_{n-1},\xi x_n,\xi x_n),G(\xi v,S(\xi v),S(\xi v)),\\ G(v,S(v),S(v)),G(\xi x_{n-1},S(\xi v),S(\xi v)),G(\xi v,S(\xi v),S(\xi v)),\\ G(\xi v,\xi x_n,\xi x_n),G(\xi x_{n-1},\xi v,S(\xi v)),G(\xi x_{n-1},\xi v,S(\xi v)),\\ G(\xi v,\xi v,\xi x_n)\end{array}\right\}}\psi(t)dt$$

$$\leq q\max\int_0^{\left\{\begin{array}{c}G(\xi x_{n-1},\xi v,\xi v),G(\xi x_{n-1},\xi x_n,S(\xi v)),G(\xi v,\xi x_n,S(\xi v)),\\ G(\xi v,S(\xi v),S(\xi v)),G(\xi x_{n-1},S(\xi v),S(\xi v)),G(\xi x_{n-1},\xi x_n,\xi x_n),\\ G(\xi v,\xi x_n,\xi x_n),G(\xi x_{n-1},\xi v,S(\xi v)),G(\xi v,\xi v,\xi x_n)\end{array}\right\}}\psi(t)dt$$

$$\leq q\max\int_0^{\left\{\begin{array}{c}G(\xi x_{n-1},\xi v,\xi v),G(\xi x_{n-1},\xi x_n,\xi x_n),\\ G(\xi v,S(\xi v),S(\xi v)),G(\xi x_{n-1},S(\xi v),S\xi(v)),\\ G(\xi v,\xi x_n,\xi x_n)\end{array}\right\}}\psi(t)dt$$

[3.1 (xi)]

Hence by taking the limit $n\to\infty$ we have the function G is continuous on its variable

Therefore

$$\int_0^{G(\xi v,S(\xi v),S(\xi v))}\psi(t)dt\ \leq\ \int_0^{G(\xi v,S(\xi v),S(\xi v))}\psi(t)dt$$

This gives contradiction. Since $0\leq q\leq {}^1\!/_2$,

Therefore $\xi v=S(\xi v)$.

Uniqueness: Let us assume that $\xi w\neq\xi v$ is such that

$T(\xi w)=\xi w$, then we have

$$\int_0^{G(v,w,w)}\psi(t)dt\ \leq\ q\max\int_0^{\{G(v,w,w),G(w,v,v)\}}\psi(t)dt$$

Then

$$\int_0^{G(\xi v,\xi w,\xi w)}\psi(t)dt\ \leq\ k\int_0^{G(\xi w,\xi v,\xi v)}\psi(t)dt$$

Now again by same procedure we will find

$$\int_0^{G(\xi w\xi,v,\xi v)}\psi(t)dt\ \leq\ q\int_0^{G(\xi v,\xi w,\xi w)}\psi(t)dt$$

thus

$$\int_0^{G(\xi w\xi,v,\xi v)}\psi(t)dt\ \leq\ q^2\int_0^{G(\xi v,\xi w\xi,w)}\psi(t)dt$$

[3.1 (xii)]

Which proves that $\xi w=\xi v$, since $0\leq q\leq {}^1\!/_2$ we say that S is continuous at ξv.

Let $\{\xi y_n\}$ be any classification of U, then

$\lim\{\xi y_n\}=\xi v$, then

$$\int_0^{G(S(\xi y_n),S(\xi v),S(\xi y_n))}\psi(t)dt\ \leq\ q\max\int_0^{\left\{\begin{array}{c}G(\xi y_n,\xi v,\xi y_n),G(\xi y_n,S(\xi y_n),S(\xi y_n)),\\ G(\xi v,S(\xi v),S(\xi v)),G(\xi y_n,S(\xi v),S(\xi v)),\\ G(\xi v,S(\xi y_n),S(\xi y_n))\end{array}\right\}}\psi(t)dt$$

[3.1 (xiii)]

And we have

$$\int_0^{G(S(\xi y_n),\xi v,S(\xi y_n))}\psi(t)dt\ \leq\ q\max\int_0^{\left\{\begin{array}{c}G(\xi y_n\xi,v,\xi y_n),G(\xi y_n,S(\xi y_n),S(\xi y_n)),\\ G(y_n,v,v)\end{array}\right\}}\psi(t)dt$$

[3.1 (ixv)]

Now by prop (G5) we have

$$\int_0^{G(\xi y_n, S(\xi y_n), S(\xi y_n))} \psi(t)dt \le q\, max \int_0^{[G(\xi y_n, \xi v, \xi v) + G(\xi v, S(\xi y_n), S(\xi y_n))]} \psi(t)dt$$

[3.1 (xv)]

And following casesaries

(i) $\int_0^{G(S(\xi y_n), \xi v, S(\xi y_n))} \psi(t)dt \le q \int_0^{G(\xi y_n, \xi y_n, \xi v)} \psi(t)dt$

(ii) $\int_0^{G(S(\xi y_n), \xi v, S(\xi y_n))} \psi(t)dt \le q \int_0^{G(\xi y_n, \xi v, \xi v)} \psi(t)dt$

(iii) $\int_0^{G(S(\xi y_n), \xi v, S(\xi y_n))} \psi(t)dt \le q \int_0^{G(\xi y_n, \xi v, \xi v)} \psi(t)dt$

In every case we get as limit $n \to \infty$ we have

$$G(\xi v, S(\xi y_n), S(\xi y_n)) \to 0$$

Hence by proposition [8.3] we have sequence $\{S(\xi y_n)\}$ is random G- convergent to ξv,
i.e. $\xi v = S(\xi v)$
and by proposition we have S is G-random continuous at v.
Following results can be proved easily at the basis of theorem 3.1

Corollary 3.2: Suppose a metric (U, G) space be G- Complete or it is Complete G-Metric Space for a mapping S : U → U which gratifies the subsequent axioms for some $m \in N$ and for all $\xi x, \xi y, \xi z \in U$.

$$\int_0^{G(S^m(\xi x), S^m(\xi y), S^m(\xi z))} \psi(t)dt$$

$$\le q\, max \int_0^{\left\{\begin{array}{c} G(\xi x, \xi y, \xi z), G(\xi x, S^m(\xi x), S^m(\xi y)), G(\xi y, S^m(\xi y), S^m(\xi z)), \\ G(\xi z, S^m(\xi z), S^m(\xi x)), G(\xi x, S^m(\xi y), S^m(\xi z)), G(\xi y, S^m(\xi x), S^m(\xi z)) \\ G(\xi z, S^m(\xi x), S^m(\xi y)), G(\xi x, S^m(\xi x), S^m(\xi x)), G(\xi y, S^m(\xi y), S^m(\xi y)), \\ G(\xi z, S^m(\xi z), S^m(\xi z)), G(\xi x, S^m(\xi y), S^m(\xi y)), G(\xi y, S^m(\xi z), S^m(\xi z)), \\ G(z, S^m(x), S^m(x)), G(x, y, S^m(z)), \\ G(\xi x, \xi z, S^m(\xi y)), G(\xi z, \xi y, S^m(\xi x)) \end{array}\right\}} \psi(t)dt$$

[3.2(i)]

Where q > 0, $0 < q < 1$ *and* $\psi : (0,1) \to (0,1), (0,1]$ is a aligning which is the sum on each compact subclass of $[0, +\infty]$, is lebesgue integrable and Non-destructive, such that

$$\int_0^{\epsilon} \psi(t)dt > 0 \;, \forall \epsilon > 0 \qquad 3.2(ii)$$

Where $q \in (0, {}^1/_2]$ then v is an exclusive invariant point of mapping S and S^m is G- unceasing at v.

Theorem 3.3 : Suppose a metric (U, G) space be G- Complete or it is Complete G-Metric Space for a mapping S : U → U which gratifies the subsequent axioms for some $m \in N$ and for all $\xi x, \xi y, \xi z \in U$

$$\int_0^{G(S(\xi x), S(\xi y), S(\xi z))} \psi(t)dt \le q\, max \int_0^{\left\{\begin{array}{c} G(\xi x, S(\xi x), S(\xi x)) + G(\xi y, S(\xi y), S(\xi x)), \\ G(\xi y, S(\xi y), S(\xi z)) + G(\xi z, S(\xi z), S(\xi y)), \\ G(\xi z, S(\xi z), S(\xi x)) + G(\xi x, S(\xi x), S(\xi z)) \end{array}\right\}} \psi(t)dt$$

[3.3(i)]

Where q > 0, $0 < q < 1$ *and* $\psi : (0,1) \to (0,1), (0,1]$ is a aligning which is the sum on each compact subclass of $[0, +\infty]$, is lebesgue integrable and Non-destructive, such that

$$\int_0^{\epsilon} \psi(t)dt > 0 \;, \forall \epsilon > 0 \qquad [3.3(ii)]$$

Where $q \in \left(0, {}^{1}/_{2}\right]$ then ξv is an exclusive invariant point of mapping S and S^m is G- unceasing at ξv.

Corrolary 3.4: Suppose a metric (U, G) space be G- Complete or it is Complete G-Metric then for a mapping S : U → U which gratifies the subsequent axioms for all $x, y, z \in U$, then

$$\int_0^{G(S^p(\xi x),S^p(\xi y),S^p(\xi z))} \psi(t)dt \;\le q\, max \int_0^{\left\{\begin{matrix} G(\xi x,S^p(\xi x),S^p(\xi x))+G(\xi y,S^p(\xi y),S^p(\xi x)), \\ G(\xi y,S^p(\xi y),S^p(\xi z))+G(\xi z,S^p(\xi z),S^p(\xi y)), \\ G(\xi z,S^p(\xi z),S^p(\xi x)),G(\xi x,S^p(\xi x),S^p(\xi z)) \end{matrix}\right\}} \psi(t)dt$$

[3.4(i)]

Where $q > 0, 0 < q < 1$ *and* $\psi : (0,1) \to (0,1), (0,1]$ is a aligning which is the sum on each compact subclass of $[0, +\infty]$, is lebesgue integrable and Non-destructive, such that

$$\int_0^{\epsilon} \psi(t)dt > 0 \;, \forall \epsilon > 0 \qquad [3.4(ii)]$$

Where $q \in \left(0, {}^{1}/_{2}\right]$ then v is an exclusive invariant point of mapping S and S^p is G- unceasing at ξv.

Theorem 3.5: Suppose a metric (U, G) space be G- Complete or it is Complete G-metric then for a mapping S : U → U which gratifies the subsequent axioms for all $\xi x, \xi y, \xi z \in U$, then

$$\int_0^{G(S(\xi x),S(\xi y),S(\xi z))} \psi(t)dt \;\le q\, max \int_0^{\left\{\begin{matrix} G(\xi z,S(\xi z),S(\xi z))+G(\xi x,S(\xi z),S(\xi z)), \\ G(\xi z,S(\xi y),S(\xi y))+G(\xi x,S(\xi y),S(\xi y)), \\ [2G(\xi y,S(\xi x),S(\xi x))],[2G(\xi z,S(\xi x),S(\xi x))] \end{matrix}\right\}} \psi(t)dt$$

[3.5(i)]

Where $q > 0, 0 < q < 1$ *and* $\psi : (0,1) \to (0,1), (0,1]$ is a aligning which is the sum on each compact subclass of $[0, +\infty]$, is lebesgue integrable and Non-destructive, such that

$$\int_0^{\epsilon} \psi(t)dt > 0 \;, \forall \epsilon > 0 \qquad [3.5(ii)]$$

Where $q \in \left(0, {}^{1}/_{3}\right]$ then v is a exclusive invariant point of mapping S and S is G- continuous at v.

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