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LATTICE REDUCTION AIDED PROBABILISTIC TREE PRUNING (PTP) FOR A SPHERE DECODER IN MIMO SYSTEMS

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Abstract

In digital communication employing Multiple Input Multiple Output (MIMO), Maximum Likelihood (ML) detection is optimum method to decode received signal vector, if channel matrix is known. ML cannot be realized as complexity increases exponentially with the increase in number of antennas and constellation size. Sphere Decoder (SD) is used to achieve the near ML performance with polynomial complexity. As many applications are modeled as integer least square problems, finding solution this problem is equivalent to finding closest lattice point in the sphere. Some of the preprocessing methods such as lattice basis reduction may be applied before sphere decoding to convert integer least square problem into simple . Among them, Lenstra, Lenstra and Lovász (LLL) is a strategic approach to lattice reduction. LLL algorithm reduces the complexity by searching through less number of paths. The paper proposes to combine PTP-SD with lattice basis reduction for complexity reduction. A Look Up Table (LUT) is prepared using Radius Choice algorithm for calculation of initial search radius for SD. LUT is used to start the search process and SD updates the search radius using the PTP algorithm. Simulations are carried out for 4 and 16-QAM over 4×4 and 8×8 MIMO configurations. The results revealed that the initial search radius of SD reduced by about 35%, the average number of Floating Point Operations (FLOPS) reduced by 50% as number of nodes visited also decreased, without degrading the performance.

Introduction

Due to the large available bandwidth on a scattering rich wireless channel [2], multiple-input multiple-output (MIMO) system has been extensively used in the communication system. MIMO uses multiple antennas at both the transmitter and receiver to improve the performance of the communication system. It has attracted attention in wireless communications, because it offers significant increase in data throughput without any increase in transmit power. It achieves this by spreading the total transmit power over the antennas which improves the search process. In this situa-tion, the first point obtained by SD is known as Babai point or Zero-Forcing Decision Feedback Equalization (ZF-DFE) point.

The radius can be updated as the distance between Babai point and the received point. Minimum Mean Square Error (MMSE) detection is used [8] to obtain the initial point. The two methods mentioned above, ensure that there is at least one point inside the sphere, but the radii are often too large due to the poor performance of ZF spectral efficiency. As the number of transmit antennas and the constellation size increases, the complexity of any decoding system increases exponentially. Maximum Likelihood Decoding (MLD) is the optimum decoding method. But the exhaustive full search makes it unrealizable in a practical system. SD is one of the methods to reduce the complexity of MLD without sacrific-ing the performance.

Choosing the initial radius of the sphere and updating its radius whenever the required lattice point is not found within the sphere, contribute to the complexity of the system. Many algorithms have been proposed to further reduce the complexity of SD such as Maximum-Likelihood detection and the search for the closest lattice point method [3], Closest point search in lattices method [4], [5], minimum mean square error (MMSE) detection method [8], Radius Choice Algorithm [1] to set the initial radius and Increasing Radii Algorithm (IRA), Probabilistic Tree Pruning Sphere Decoding (PTP-SD) algorithms for updating the radius. The number of visited nodes determines the complexity of SD. This can be reduced by removing the unlikely branches in early stage of sphere search. The sphere constraint of the SD algorithm offers a loose necessary condition in the early layers of search. In [3] and [4] they choose the initial radius for DFE and MMSE. Hence, it does not reduce the complexity of SD considerably. In this paper, the PTP-SD algorithm uses the radius selection algorithm to determine the initial search radius. It is proposed that the complexity of the Sphere Decoding can be reduced further by combining the PTP-SD algorithm with the radius choice algorithm.

The PTP-SD algorithm uses the radius selection algorithm to determine the initial search radius which is obtained from the LUT generated. It is shown that a combination of these two algorithms will lead to the significant reduction in the complexity by maintaining the same performance. Besides, the proposed method does not add any additional complexity [1] to SD. The other advantage is that the radii can be calculated and stored in a table in advance. A particular radius can be obtained in the stage of preprocessing by looking up the table according to the SNR at any instant of time.

A powerful preprocessing technique for improving the performance of suboptimum data detectors is lattice reduc-tion (LR) [17], [18]. The idea behind LR [15], [16] is to transform the problem into a domain where the effective channel matrix is better conditioned than the original one. The channel realization is regarded as a basis of a lattice, and one attempts to find a better (i.e., more orthogonal) basis for the same lattice. Suboptimum detectors can then be applied to this better basis, which results in improved performance. So far, almost exclusively the LLL algorithm [19] has been considered for LR-assisted data detection. The LLL algorithm allows suboptimum detectors to exploit all of the available diversity [20].

Since the reduced basis has better mathematical properties like smaller orthogonality defect, smaller condition number, etc. Solving detection and precoding problems w.r.t the re-duced basis offers advantages with respect to performance and complexity. For example, it was shown recently that in some scenarios even suboptimum detection/precoding tech-niques can achieve full diversity when preceded by LLL lattice reduction as in [21–24].

Generally speaking, LLL algorithm is able to reduce the computational complexity of sphere decoding by reducing the norm of upper triangular matrix R of channel matrix H and by reducing the total number of search paths. The tree representation of sphere decoding, LLL algorithm can shrink the integer interval at each level of the tree. Therefore the number of nodes is reduced at each level of the tree. Conse-quently, the search path as well as the complexity of sphere decoding is reduced.

The remainder of the paper is organized as follows. In section II, the system model is introduced. In section III, the Radius Choice algorithm with PTP-SD algorithm is ex-plained, LLL is outlined in section IV. Section V shows with result analysis of the proposed method and section VI pro-vides the conclusions.

System model

A. Sphere Decoder

Considering an uncoded MIMO system with M transmit and N receive antennas (M N), the received complex signal at each instant time is given by,

$$y_c = \left(1/\sqrt{ME}\right) H_c x_c + n_c \tag{1}$$

Where x_c is the transmitted symbol vector whose components are elements of a Quadrature Amplitude Modulation (QAM) signal set $\mathcal{O}_A$ with size A. It is assumed that all vectors are transmitted with the same probability. H_c is a complex channel matrix known perfectly to the receiver. n_c is a circular symmetric complex Gaussian noise vector. E is the average power of the transmitted symbol. If the signal to noise ratio is ρ, the variance of the component of n_c is 1/ρ.

In order to use SD, the complex number signal model in (1) needs to be reformulated to a real number signal model as follows.

$$y = \begin{bmatrix} \Re(y) \\ \Im(y) \end{bmatrix}$$

$$x = \begin{bmatrix} \Re(x_c) \\ \Im(x_c) \end{bmatrix}$$

$$v = \begin{bmatrix} \Re(v_c) \\ \Im(v_c) \end{bmatrix}$$

$$H = \left(1/\sqrt{ME}\right) \begin{bmatrix} \Re(H_c) - \Re(H_c) \\ \Im(H_c) \Re(H_c) \end{bmatrix} \tag{2}$$

Where, $\Re(\cdot)$ and $\Im(\cdot)$ are the real and imaginary parts of its argument. Then the real number signal model is given by

$$y = Hx + n \tag{3}$$

Let m = 2 M, n = 2 N, hence, H is a (n × m) real matrix. The real MLD is given by,

$$\hat{x} = \arg\min |y - Hx|^2 \tag{4}$$

SD reduces the complexity by limiting the search space in a hypersphere $S(y, \sqrt{C})$ centered at y, where C is the squared radius of the sphere. SD can be expressed as,

$$|y - Hx|^2 \le C \tag{5}$$

Performing QR-decomposition of H as $H = [QQ'][R0^T]$, where R is an m × n upper triangular matrix with positive diagonal elements, 0 is a zero matrix, Q and Q' are an n × m and n × (n–m) unitary matrices respectively. The inequality (5) is equivalent to,

$$\left| [QQ'] - \begin{bmatrix} R \\ 0 \end{bmatrix} x^2 \right| \tag{6}$$

$$|Q^T y - Rx|^2 \le C - |Q^T y|^2 \tag{7}$$

$$\lceil y' - Rx \rceil^2 \le c_0 \tag{8}$$

Where $y' = Q^T y$ and $c_0 = C - |(Q')^T y|^2$

The sphere radius of d and centered at y can be defined as,

$$X = \{x, \|Hx - y\| \le d\} \tag{9}$$

Whose condition is equivalent to,

$$\|Rx - \hat{y}\|_2^2 \le d^2 \tag{10}$$

Where $\hat{d}^2 = d^2 - \|Q_2^T y\|_2^2$

Since R is upper triangular, so rewriting the above condition, in entry wise as,

$$\hat{d}^2 \ge \left(\sum_{i=1}^{m} (\sum_{j=1}^{m} r_{i,j} x_j - \hat{y}_1)\right)^2 \tag{11}$$

Where, $r_{i,j}$, $j \ge i$ denotes the *(i,j)*[th] entry of *R*. The above equation is expanded to get the equations.

$$\hat{d}^2 \ge \left(\hat{y}_m - r_{m,m}x_m\right)^2 + \left(\begin{matrix}\hat{y}_{m-1} - r_{m-1,m-1}x_m \\ -r_{m-1,m-1}x_m\end{matrix}\right)^2 + \cdots \quad (12)$$

The first term in the right side of above equation depends only on the m[th] entry x_m of lattice point *s*, the second term depends on the entries x_m and x_{m-1}, and so on. By solving, we get,

$$\left\lceil \frac{-\hat{d} + \hat{y}_m}{r_{m,m}} \right\rceil \le x_m \le \left\lfloor \frac{\hat{d} + \hat{y}_m}{r_{m,m}} \right\rfloor \quad (13)$$

Following the above procedure, the intervals for x_{m-1}, x_{m-2} etc. are obtained until x_1 is reached. Then it is able to determine all the lattice points in the hyper sphere of radius.

In the below figure the numbers labeled for each node are the path metrics. Note that the dotted nodes are skipped since they are outside of sphere constraint.

Initial Radius selection and updating

A. Expected number of points in sphere using Radius choice algorithm

In Radius Choice algorithm, the initial radius can be obtained corresponding to the expected number of points for particular values of SNR. The received symbol vector is denoted as $\tilde{x}$ and the actual transmitted symbol vector as x

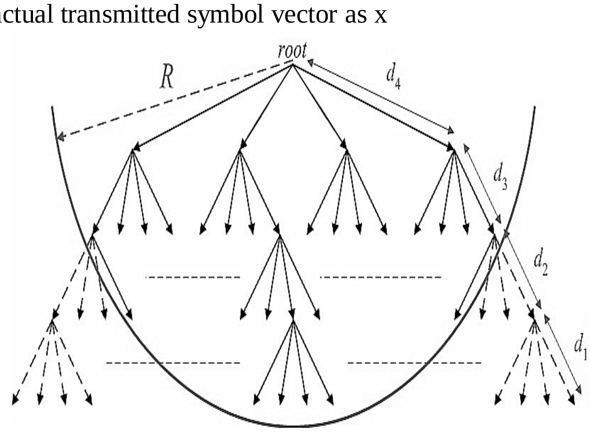

Figure 1. Illustration of sphere decoding in a tree.

Then,

$$y - H\tilde{x} = Hx + n - H\tilde{x} = H(x - \tilde{x}) + n = He + n \quad (14)$$

Where $e = x - \tilde{x}$ is an error symbol vector. Therefore, the components of $\left|y - H\tilde{x}\right|$ are i.i.d. $\overline{N}\left(0, \sigma^2 + \sigma_h^2|e|\right)$ random variables and $\left|y - H\tilde{x}\right|^2$ is a scaled chi - squared distribution with m degrees of freedom, where σ_h^2 is the variance of the component of *H*. When the decoding is perfect, $\tilde{x}$ equals to *x*, so $y - Hx = n$ is Gaussian random vector whose component is $\overline{N}(0,\sigma^2)$ random variable. Then when a definite radius *C* is given, we can obtain the probability that the lattice point $\hat{x}$ is in the sphere,

$$F_{\hat{x}}(C) = \int_0^{(C/\sigma^2+\sigma_h^2|e|^2)} \left(1/2^{1/2}\Gamma(m/2)\right)\left(u^{\frac{m}{2}-1}\right)e - u/2\,du \quad (15)$$

$$F_{\hat{x}}(C) = \Phi(C/\sigma^2 + \sigma_h^2|e|^2) \quad (16)$$

Where, σ^2 is the co-variance and σ_h^2 is the covariance of component of channel matrix *H*.

$$\sigma^2 = M\left(L^2 - 1\right)/6\rho \quad (17)$$

Where, L^2 is the QAM constellation, $\Gamma(\cdot)$ is the Gamma function and $\Phi(\cdot)$ is the Cumulative Distributive Function (CDF) of chi-square distribution. Here, form a table of initial radius value for any expected number of lattice points for a given value of SNR. A sequence of number of points such as *D1, D2, D3* and so on are considered with constant incremental steps. Then radius values *C1, C2, C3* and so on respectively using the following equations (15), (16) and (18) for a given value of SNR is calculated. For 16-QAM, the equation for the expected number of points is given by,

$$D(C) = \sum_{i=0}^{m} \binom{m}{l} F_{\tilde{x}}(C) \quad (18)$$

For 16-QAM, the equation for the expected number of points is given by,

$$D(C) = \left(1/2^m\right)\sum_q \sum_{i=0}^{m} \binom{m}{l} g_{kl}(q) F_{\tilde{x}}(C) \quad (19)$$

Where, $g_{kl}(q)$ is the coefficient of x^q in the polynomial $(1+x+x4+x9)^l\ (1+2x+x4)^{k-l}$. Similar results can be obtained for 64-QAM and other constellations. The initial radius C1 is chosen such that it should eliminate the too-large and the too-small conditions. The too-large condition implies that there are many points within the sphere. Hence, the complexity cannot be reduced effectively. The too small condition implies that there is no lattice point within the sphere which leads to repetitive search and hence, increases the complexity. If the search fails with C1, then we start the new search with C2 as the initial radius. If there is only one lattice point then the solution will be the ML solution.

In this paper, it is proposed that the complexity of the PTP-SD can be reduced further by combining the SD algorithm with the radius choice algorithm and performing Lattice reduction using LLL algorithm. In PTP-SD algorithm, instead of starting the search radius from infinity, the points can be searched from the initial radius which is obtained from the LUT. Here it is shown that the complexity reduces further with LR techniques, maintaining the same performance.

B. Probabilistic Tree Pruning-SD

Inputs: R, where R is upper triangular matrix , $\hat{y}$ is the y reduced by the QR decomposition. d, radius of sphere.

Output: x or null.

Step 1: set $k=m$, $d_m^2 = d^2 - \left\|Q_2^T y\right\|^2, \hat{y}_{m|m+1} = y_m$

Step 2: (Bounds for x_k),

set $UB_{x(k)} = \frac{d_k + \hat{y}_{k|k+1}}{r_{k,k}}$, $LB_{x(k)} = \frac{-d_k + \hat{y}_{k|k+1}}{r_{k,k}}$

Step 3: (Increase x_k) $x_k = x_k + 1$

If $LB_{x(k)} \leq UB_{x(k)}$, go to step 5; else go to step 4.

Step 4: (Increase k) $k=k+1$

If $k=m+1$, terminate algorithm, else go to step 3.

Step 5: (Decrease k) if $k=1$, go to Step 6; else

$k=k-1$, $\hat{y}_{k|k-1} = \hat{y}_k - \sum_{j=k+1}^{m} r_{i,j} x_j$,

$d_k^2 = d_{k+1}^2 - \left(\hat{y}_{k+1|k+2} - r_{k+1,k+1} x_{k+1}\right)^2$

and go to step 2.

Step 6: solution found. Save x and its distance from y,

$d_m^2 = d_1^2 + \left(y_1 - r_{1,1} x_1\right)^2$

and go to step 3.

Lattice basis reduction

Some of the preprocessing methods such as QR decomposition and lattice reduction may be applied before sphere decoding to transform the integer least squares problem into a simpler form. In this approach, it is an attempt to find an invertible $m \times m$ matrix M, such that both M and M^{-1} are integer matrices (unimodular matrices), and therefore the matrix $H\ M$ preserves the lattice structure. Denote $s = M\ t$ and $G = H\ M$, where M is aforementioned $m \times m$ invertible integer matrix (unimodular matrix), then the integer least squares problem in equation (10) becomes

$$\min_{t \varepsilon Z^m} \left\|Gt - y\right\|_2^2 \qquad (20)$$

Thus, sphere decoding may be applied to equation (20), then it is straightforward to solve x by $x = Mt$. However, the lattice reduction approach is itself NP-hard; the famous LLL algorithm is a strategic approach to lattice reduction. The LLL algorithm is widely used by researchers as a preprocessor to solve the integer least squares problem, it is often arguably referred to as an integer Gram-Schmidt procedure. Suppose that QR decomposition is applied to the integer least squares problem equation (18) reduces to

$$\min_{x \varepsilon Z^m} \left\|Rx - \hat{y}\right\|_2^2 \qquad (21)$$

Then, apply the LLL algorithm to the upper triangular matrix R to decompose R into

$$R = \hat{Q}\hat{R}M^{-1} \qquad (22)$$

Where, $\hat{Q}$ is orthogonal, $\hat{R}$ is upper triangular and M is unimodular, so M^{-1} is an integer matrix. The LLL algorithm transforms a basis formed by the columns of R into a basis formed by the columns of $\hat{R}$, the lengths of the columns of $\hat{R}$ are shorter than those of R, so that the columns of $\hat{R}$ form a reduced basis for a lattice space.

A. LLL algorithm

The algorithm inputs are the basis matrix H and the reduction parameter δ .

1. $[Q, R]$ qr(H)
2. $l \leftarrow 2$
3. $H_l \leftarrow H_l - \left[\frac{R_{l-1,l}}{R_{l-1,l-1}}\right] H_{l-1}$
4. if $\delta \left|R_{l-1,l-1}\right|^2 \geq \left|R_{l,l}\right|^2 + \left|R_{l-1,l}\right|^2$ then
5. $H_{l-1} \leftarrow H_l$
6. $l \leftarrow \max(l-1, 2)$
7. else
   for $k = l\text{-}1$ to 1
8. $H_l \leftarrow H_l - \left[\frac{R_{k,l}}{Rk,k}\right] Hk$
9. $l=l+1$
10. end if

11. $[Q, R]$ qr(H)

LLL algorithm is able to reduce the computational complexity of sphere decoding in two ways. First, it can reduce the initial radius of the hypersphere by reducing the norm of R. Second, since sphere decoding is a depth-first search algorithm for the lattice points inside a hypersphere, the LLL algorithm can reduce the total number of search paths. Because in the tree representation of sphere decoding, LLL algorithm can shrink the integer interval at each level of the tree, therefore the number of nodes is reduced at each level of the tree. Consequently the complexity of sphere decoding is reduced.

Simulation results

In this section, we present the results of simulations for different configurations of systems.

A. LUT for initial radius using Radius Choice Algorithm

Table I is the look up tables for initial radius for the combination of expected number of points and the given value of SNR. Table I and II are generated using the expression (15), (16) and (18). Here it can be seen that D_1 is much less than D_2 especially when SNR is high while the difference between two adjacent D_i' s for $i>1$ is very small at the entire SNR regime. Here it can be observed that, as the SNR increases the initial radius, from where the search has to be started, decreases. It can also be seen that the initial search radius increases with number of antennas.

Table 1.Initial radius Look Up Table for 8 X 8 MIMO with 16-QAM when D=1, 2 …and 5.

SNR	D=1	D=2	D=3	D=4	D=5
1	51.3389	2.76026	2.93182	3.06126	3.16642
2	25.6976	2.64259	2.80683	2.93076	3.03143
3	17.1317	2.60336	2.76517	2.88726	2.98644
4	12.8347	2.58375	2.74433	2.86550	2.96394
5	10.2452	2.57198	2.73184	2.85245	2.95044
6	8.50953	2.56414	2.72350	2.84375	2.94144
7	7.26169	2.55859	2.71755	2.83754	2.93501
8	6.31876	2.55437	2.71309	2.83288	2.93019
9	5.57911	2.55106	2.70962	2.82925	2.92644
10	4.9815	2.54845	2.70684	2.82635	2.92344

B. Complexity analysis of PTP-SD with radius choice algorithm and LLL

Fig 2 is a plot of number of FLOPS vs. SNR for PTP-SD with Radius Choice and lattice reduction. Here the reduction factor δ is selected as 0.25 for LLL algorithm. The simulations are carried out for a 4×4 and 8×8 MIMO and for the constellation size of 4-QAM and 16-QAM. Fig. 3 is a plot of % reduction in average number of FLOPS Vs SNR for 8 X 8 MIMO and 16-QAM. Fig. 2 reveals that the number of FLOPS for PTP-SD with Radius Choice algorithm and LLL lattice basis reduction are reduced as compared mere PTP-SD. At higher SNR i.e. above 5 dB the reduction in the required number of FLOPS is around 50 %.

Fig. 4 is a plot of average number of nodes visited Vs SNR for 8 X 8 MIMO and 16-QAM. The graph shows that

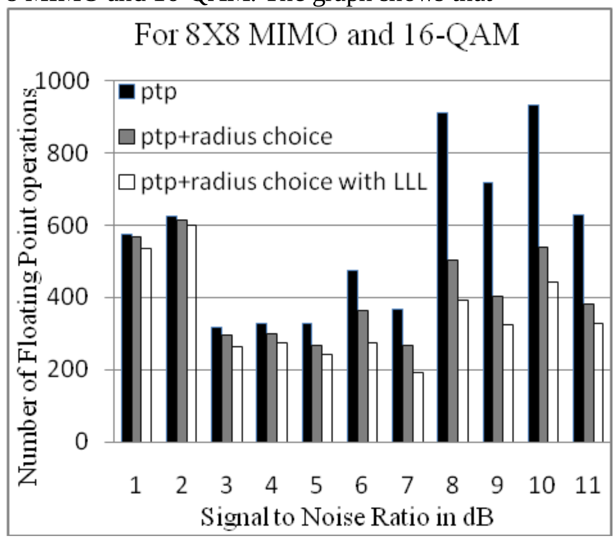

Figure 2. The plot of number of FLOPS Vs SNR for 8 X 8 MIMO and 16-QAM.

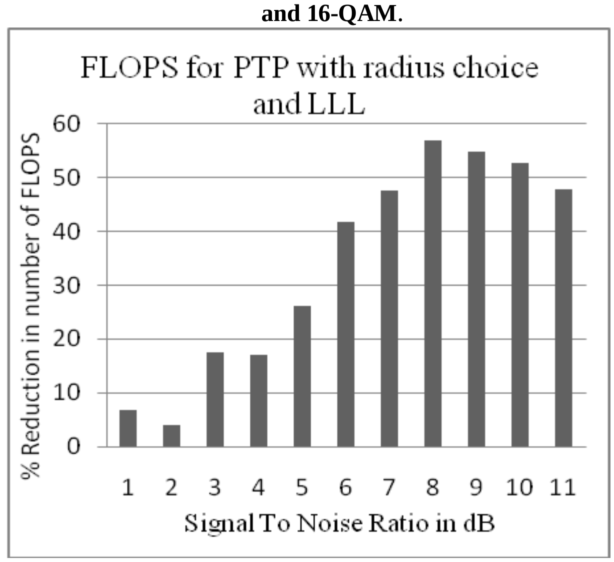

Figure 3. The plot of % reduction in average number of FLOPS Vs SNR for 8 X 8 MIMO and 16-QAM.

the number of visited nodes reduced using the lattice reduction technique. The graph is plotted over a range of SNRs from 1dB to 10dB. The reduction in the visited nodes in turn reduces the search paths. Hence leads to the reduction in the complexity.

Fig. 5 is a plot of % reduction in initial search radius Vs SNR for 8 X 8 MIMO and 16-QAM. The initial search radius reduces by around 35% at higher SNR values i.e. above 5 dB and it remains almost flat from 0 to 5 dB. This gives reduced complexity of a sphere decoder with radius choice algorithm and LLL lattice basis reduction.

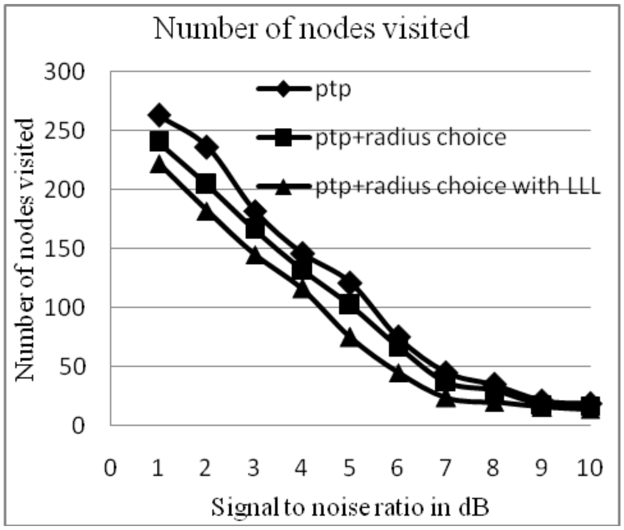

Figure 4. The plot of average number of nodes visited Vs SNR for 8 X 8 MIMO and 16-QAM

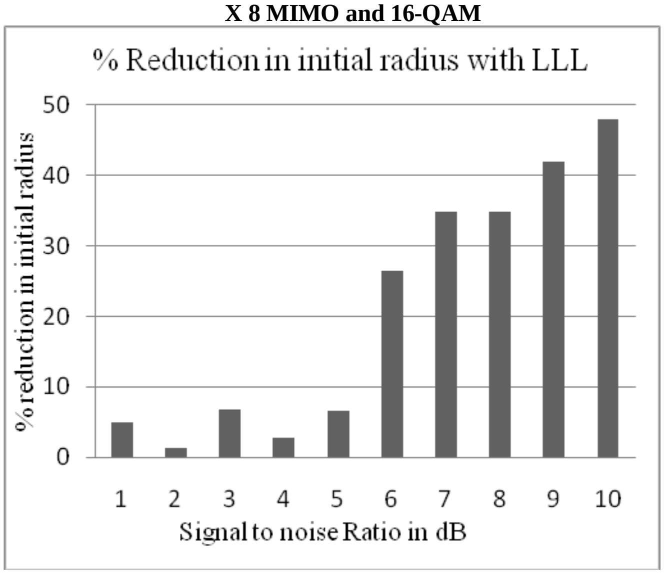

Figure 5. The plot of % reduction in initial search radius Vs SNR for 8 X 8 MIMO and 16-QAM.

Conclusions

Sphere Decoder (SD) is used to achieve the near ML performance with polynomial complexity. Finding solution to this integer least square problem is equivalent to finding closest lattice point in the sphere. The preprocessing methods such as lattice basis reduction are applied before sphere decoding to convert integer least square problem into simple one. LLL algorithm reduces the complexity by searching through less number of paths. Here a combination of PTP-SD with lattice basis reduction for complexity reduction outperforms mere PTP-SD. A Look Up Table (LUT) is generated using Radius Choice algorithm for calculation of initial search radius for SD. LUT is used to start the search process and SD updates the search radius using the PTP algorithm. Here the simulations are carried out for 4 and 16 QAM over 4×4 and 8×8 MIMO configurations. The results revealed that the initial search radius of SD reduces by around 35%, the average number of Floating Point Operations (FLOPS) reduces by 50%, above 5dB. The performance is unaltered until 5dB. Consequently the number of nodes visited also decreased, in turn reducing the complexity, without degrading the performance.

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