International Journal of Advanced Technology & Engineering Research (IJATER)
International Conference on "Recent Advancement in Science & Technology" (ICRAST 2017)

COUPLED FIXED POINT THEOREMS IN FUZZY METRIC SPACES

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Abstract

The aim of the paper is to obtain coupled fixed point theorems for mapping having the mixed monotone property in partially ordered non-Archimedean complete fuzzy metric spaces.

Keywords: Fuzzy Metric Spaces, Mixed monotone property, Partially ordered set, Coupled fixed point theorems.

Mathematics Subject Classification: 52H25, 47H10.

Introduction

Fixed point theory is one of the most successful and effective tools in Mathematics which has enormous applications within as well as outside the Mathematics. Even though noted improvements in computer skill and its remarkable success in facilitating many areas of research, there still stands one major short coming: computers are not designed to handle situations where in uncertainties are involved. To deal with uncertainty, we need techniques other than classical ones where in some specific logic is required. Fuzzy set theory is one of the uncertainty approaches where in topological structures are basic tools to develop mathematical models compatible to concrete real life situations.

The fundamental work for the fuzzy theory was first given by Zadeh [14] in 1965, who introduced the concept of fuzzy set. Kramosil and Michalek [7] developed the fuzzy metric space and later George and Veeramani [4] modified the notion of fuzzy metric spaces by introducing the concept of continuous t–norm. Many researchers have extremely developed the theory by defining different concepts and amalgamation of many properties.

Bhaskar and Lakshmikantham [2],Nieto and Rodriguez-Lopez [10,11],Ran and Reurings [12] and Agarwal et al.[1] presented some new results for contractions in partially ordered metric spaces.

Bhaskar and Lakshmikantham[2] brought with the concepts of coupled fixed points and mixed monotone property and derive the results by proving the existence and uniqueness of the solution for a periodic boundary value problem. Later on these results were extended and generalized by Sedghi et al. [13], Fang [3] and Xin-Qi Hu [5] etc.

Dorel Mihet [9] proved a common fixed point theorem in Non- Archimedean fuzzy metric space. The intent of this paper is to obtain new coupled fixed point theorems for mappings having the mixed monotone property in partially ordered non-Archimedean complete fuzzy metric spaces.

Preliminaries

Definition 2.1:[8] A mapping $*$: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is called a continuous t-norm if $*$ is satisfying the following conditions:

1) $*$ is commutative and associative;
2) The mapping $*$: $[0, 1]\times [0, 1]\rightarrow [0, 1]$ is continuous.
3) $a * 1= a$ for all $a\in[0, 1]$;
4) $a* b \leq c *d$ whenever $a \leq c$ and $b \leq d$ for all $a ,b,c ,d \in[0, 1]$.

Definition 2.2:[8] The 3-tuple $(X,M, *)$ is called a non-Archimedean fuzzy metric space if X is an arbitrary non empty set , $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times [0, \infty)$ such that the followings conditions hold:

1) $M(x, y, 0) = 0$,
2) $M(x,y,t)=1$,for all $t > 0$ if and only if $x=y$,
3) $M(x,y,t)= M(y,x,t)$,
4) $M(x,y,t) * M(y,z,s) \leq M(x,z, \max\{t, s\})$ for all $x, y, z \in X$ and $t,s > 0$,
5) $M(x,y,.)$: $[0, \infty)\rightarrow[0, 1]$ is left continuous.

Definition 2.3: A Partially ordered set is a set P and a binary relation $\leq$,denoted by (X,≤),
Such that for all a, b, c ∈ P,

1) a ≤ a (reflexivity),
2) a ≤ b and b ≤ c implies a ≤ c (transitivity),
3) a ≤ b and b ≤ a implies a=b (anti-symmetry).

Definition 2.4: [6] An element (x,y) ∈ X × X is a called a coupled fixed point of the mapping F:X × X→ X if x=F(x,y) and y=F(y,x).

Definition 2.5: [6] Let (X,≤) be a partially ordered set and F: X × X→ X. The mappings F is said to has the mixed monotone property if F(x, y) is monotone non decreasing in x and is monotone non-increasing in y, that is, for any x, y ∈ X

$x_1, x_2 \in X, x_1 \leq x_2 \Rightarrow F(x_1, y) \leq F(x_2, y)$
and
$y_1, y_2 \in X, y_1 \leq y_2 \Rightarrow F(x, y_1) \geq F(x, y_2)$

Definition 2.6: [6] Let (X, M, ∗) be a non-archimedean fuzzy metric space. A sequence $\{x_n\}$ in X is said to converge to x ∈ X if and only if

$\lim_{n\to\infty} M(x_n, x, t) = 1$ for each t<0.

A sequence $\{x_n\}$ in X is called Cauchy sequence if and only if

$\lim_{n\to\infty} M\left(x_n, x_{n+p}, t\right) = 1$ for each t<0 and p = 1,2,3,…….

A non-Archimedean fuzzy metric space (X, M, ∗) is said to be complete if and only if every Cauchy sequence in X is convergent in X.

Main Results

Theorem 3.1:
Let (X, ≤) be a partially ordered set and (X, M, ∗) is a complete non-archimedean fuzzy metric space. Let F: X × X→ X be a continuous mapping having the mixed monotone property on X such that there exits two elements $x_0, y_0 \in X$ with $x_0 \leq F(x_0, y_0)$ & $y_0 \geq F(y_0, x_0)$ and

$$M(F(x,y), F(u,v), kt) \geq \max\{M(F(u,v), x, t) * M(F(u,v), u, t), M(F(x,y), x, t) * M(F(x,y), u, t)\}$$

(3.1.1)
for all x, y, u, v ∈ X with x ≥ u , y ≤ v and k ∈ (0,1) then F has a coupled fixed point in X.

Proof: Let $x_0, y_0 \in X$ with $x_0 \leq F(x_0, y_0)$ and $y_0 \geq F(y_0, x_0)$ (3.1.2)
Define the sequences $\{x_n\}$ and $\{y_n\}$ in X such that,
$x_{n+1} = F(x_n, y_n)$ and $y_{n+1} = F(y_n, x_n)$ for all n=0,1,2,……… (3.1.3)
We claim that $\{x_n\}$ is monotonic increasing and $\{y_n\}$is monotonic decreasing
i.e.,
$x_n \leq x_{n+1}$ and $y_n \geq y_{n+1}$ for all n=0,1,2,……… (3.1.4)
from (3.1.2) and (3.1.3),we have
$x_0 \leq F(x_0, y_0)$ and $y_0 \geq F(y_0, x_0)$ and $x_1 = F(x_0, y_0)$ and $y_1 = F(y_0, x_0)$.
thus $x_0 \leq x_1, y_0 \geq y_1$,i.e. (3.1.4) is true for n=0.
Now suppose that (3.1.4) hold for some n.
i.e. $x_n \leq x_{n+1}$ and $y_n \geq y_{n+1}$
We shall prove that (3.1.4) is true for n+1.
Now $x_n \leq x_{n+1}$and $y_n \geq y_{n+1}$
then by mixed monotone property of F,
we have

$$x_{n+2} = F(x_{n+1}, y_{n+1}) \geq F(x_n, y_{n+1}) \geq F(x_n, y_n) = x_{n+1}$$
and
$$y_{n+2} = F(y_{n+1}, x_{n+1}) \leq F(y_n, x_{n+1}) \leq F(y_n, x_n) = y_{n+1}$$

Thus by mathematical induction principle (3.1.4) holds for all n in N. So,

$$x_0 \leq x_1 \leq x_2 \leq \dots x_n \leq x_{n+1} \leq \dots$$
and
$$y_0 \geq y_1 \geq y_2 \geq \dots y_n \geq y_{n+1} \geq \dots$$

Since, $x_{n-1} \leq x_n$ and $y_{n-1} \geq y_n$,
Put $x = x_n , y = y_n, u = x_{n-1}, v = y_{n-1}$in (3.1.1) we have

$$M(F(x_n, y_n), F(x_{n-1}, y_{n-1}), kt) \geq \max\{M(F(x_{n-1}, y_{n-1}), x_n, t) * M(F(x_{n-1}, y_{n-1}), x_{n-1}, t), M(F(x_n, y_n), x_n, t) * M(F(x_n, y_n), x_{n-1}, t)\}$$

$$M(x_{n+1}, x_n, kt) \geq \max\{M(x_n, x_n, t) * M(x_n, x_{n-1}, t), M(x_{n+1}, x_n, t) * M(x_{n+1}, x_{n-1}, t)\}$$
$$= \max\{1 * M(x_n, x_{n-1}, t), M(x_{n+1}, x_n, t) * M(x_{n+1}, x_{n-1}, t)\}$$
$$= \max\{M(x_n, x_{n-1}, t), M(x_{n+1}, x_n, t) * M(x_{n+1}, x_{n-1}, t)\}$$
$$\geq M(x_n, x_{n-1}, t)$$
$$\Rightarrow M(x_{n+1}, x_n, t) \geq M\left(x_n, x_{n-1}, \frac{t}{k^1}\right)$$
$$\geq M\left(x_{n-1}, x_{n-2}, \frac{t}{k^2}\right)$$

$\geq \mathrm{M}\left(x_{n-2}, x_{n-3}, \frac{t}{k^3}\right)$

⋮

⋮

$\geq \mathrm{M}(x_{n-(n-1)}, x_{n-n,} \frac{t}{k^n})$

$\geq \mathrm{M}(x_1, x_{0,} \frac{t}{k^n})$

$\Rightarrow \lim_{n\to\infty} \mathrm{M}(x_{n+1}, x_n, \mathrm{t}) = 1.$

We now verify that $\{x_n\}$ is a Cauchy Sequence. So,

$\mathrm{M}\,(x_n, x_{n+p,}\,\mathrm{t}) \geq \mathrm{M}(x_n, x_{n+1}, \mathrm{t}) * \cdots * \mathrm{M}(x_{n+p-1}, x_{n+p}, \mathrm{t})$

$\Rightarrow \lim_{n\to\infty} \mathrm{M}\left(x_n, x_{n+p}, \mathrm{t}\right) \geq 1 * \cdots * 1 = 1$

$\Rightarrow \{x_n\}$ is a Cauchy sequence.

Similarly, Since $y_{n-1} \geq y_n$ and $x_{n-1} \leq x_n$

Put $x = y_{n-1}$, $y = x_{n-1}, u = y_n, v = x_n$ in (3.1.1) we have

$\mathrm{M}(\mathrm{F}(y_{n-1}, x_{n-1}), \mathrm{F}(y_n, x_n), \mathrm{kt}) \geq$
$\max\{\mathrm{M}(\mathrm{F}(y_n, x_n), y_{n-1}, \mathrm{t}) *$
$\mathrm{M}(\mathrm{F}(y_n, x_n), y_n, \mathrm{t}), \mathrm{M}(\mathrm{F}(y_{n-1}, x_{n-1}), y_{n-1}, \mathrm{t}) *$
$\mathrm{M}(\mathrm{F}(y_{n-1}, x_{n-1}), y_n, \mathrm{t})\}$

$\mathrm{M}(y_n, y_{n+1}, kt)$
$\geq \max\{\mathrm{M}(y_{n+1}, y_{n-1}, t) * \mathrm{M}(y_{n+1}, y_n, t), \mathrm{M}(y_n, y_{n-1}, t) * \mathrm{M}(y_n, y_n, t)\}$

$= \max\{\mathrm{M}(y_{n+1}, y_{n-1}, t) * \mathrm{M}(y_{n+1}, y_n, t), \mathrm{M}(y_n, y_{n-1}, t) * 1\}$

$= \max\{\mathrm{M}(y_{n+1}, y_{n-1}, t) * \mathrm{M}(y_{n+1}, y_n, t), \mathrm{M}(y_n, y_{n-1}, t)\}$

$\geq \mathrm{M}(y_{n-1}, y_n, t)$

$\Rightarrow \mathrm{M}\,(y_n, y_{n+1}, t) \geq \mathrm{M}\left(y_{n-1}, y_n, \frac{t}{k^1}\right)$

$\geq \mathrm{M}\left(y_{n-2}, y_{n-1}, \frac{t}{k^2}\right)$

$\geq \mathrm{M}\left(y_{n-3}, y_{n-2}, \frac{t}{k^3}\right)$

⋮

⋮

$\geq \mathrm{M}(y_{n-n,}\, y_{n-(n-1)}\, \frac{t}{k^n})$

$\geq \mathrm{M}(y_{0,}\, y_1\, \frac{t}{k^n})$

$\Rightarrow \lim_{n\to\infty} \mathrm{M}(y_n, y_{n+1}, \mathrm{t}) = 1.$

We now verify that $\{y_n\}$ is a Cauchy sequence .So,

$\mathrm{M}\,(y_n, y_{n+p,}\,\mathrm{t}) \geq \mathrm{M}(y_n, y_{n+1}, \mathrm{t}) * \cdots * \mathrm{M}(y_{n+p-1}, y_{n+p}, \mathrm{t})$

$\Rightarrow \lim_{n\to\infty} \mathrm{M}\left(y_n, y_{n+p}, \mathrm{t}\right) \geq 1 * \cdots * 1 = 1$

$\Rightarrow \lim_{n\to\infty} \mathrm{M}\left(y_n, y_{n+p}, \mathrm{t}\right) = 1.$

Now it is clear that $\{x_n\}$ and $\{y_n\}$ are Cauchy sequence in X.

Since X is a complete fuzzy metric space, there exist x, y ∈ X such that

$\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$.

Thus by taking limit as $n \to \infty$ in equation (3.1.3) we get

$$x = \lim_{n\to\infty} x_n = \lim_{n\to\infty} \mathrm{F}\left(x_{n-1,} y_{n-1}\right) = \mathrm{F}(\lim_{n\to\infty} x_{n-1}, \lim_{n\to\infty} y_{n-1}) = \mathrm{F}(\mathrm{x}, \mathrm{y})$$

and

$$y = \lim_{n\to\infty} y_n = \lim_{n\to\infty} \mathrm{F}\left(y_{n-1,} x_{n-1}\right) = \mathrm{F}(\lim_{n\to\infty} y_{n-1}, \lim_{n\to\infty} x_{n-1}) = \mathrm{F}(\mathrm{y}, \mathrm{x}).$$

Therefore x=F(x,y) and y=F(y,x).

Hence F has a coupled fixed point.

Theorem 3.2:

Let (X, ≤) be a partially ordered set and (X,M,∗)is a complete non-archimedean fuzzy metric space . Let F:X × X→ X be a continuous mapping having the mixed monotone property on X such that there exits two elements x_0 , y_0 ∈ X with $x_0 \leq \mathrm{F}(x_0, y_0)$ & $y_0 \geq \mathrm{F}(y_{0,}\, x_0)$

Suppose that

$\mathrm{M}(\mathrm{F}(\mathrm{x}, \mathrm{y}), \mathrm{F}(\mathrm{u}, \mathrm{v}), \mathrm{kt}) \geq \varphi(\mathrm{M}(\mathrm{x}, \mathrm{u}, \mathrm{t}))$ (3.2.1)

for all x, y, u, v ∈ X with x ≥ u ,y ≤ v and k ∈ (0,1) & φ:[0,1]→ [0,1] such that $\varphi(t) \geq$ t.

Also suppose that either

(a) F is a continuous or
(b) X has the following property:
   (i) If a non decreasing sequence $\{x_n\}\to x$ then $x_n \leq x$ for all n.
   (ii) If a non decreasing sequence $\{y_n\}\to y$ then $y \leq y_n$ for all n.

Then there exists x, y ∈ X such that

x=F(x,y) and y=F(y,x)

that is F has a coupled fixed point in X.

Proof: Let x_0, y_0 ∈ X with

$x_0 \leq \mathrm{F}\;(x_0, y_0)$ and $y_0 \geq \mathrm{F}(y_{0,}\; x_0)$ (3.2.2)

Define the sequences $\{x_n\}$ and $\{y_n\}$ in X such that,

x_{n+1}= F (x_n, y_n) and y_{n+1} = F (y_n, x_n) for all n=0,1,2,……… (3.2.3)

We claim that $\{x_n\}$ is monotonic increasing and $\{y_n\}$is monotonic decreasing

i.e.,

$x_n \leq x_{n+1}$ and $y_n \geq y_{n+1}$ for all n=0,1,2,……… (3.2.4)

from (3.2.2) and (3.2.3),we have

$x_0 \leq$ F (x_0, y_0) and $y_0 \geq$ F$(y_{0,}\, x_0)$ and x_1 =F(x_0, y_0) and y_1 =F$(y_{0,}\, x_0)$.

Thus $x_0 \leq x_1, y_0 \geq y_1$, i.e. (3.2.4) is true for n=0.

Now suppose that equation (3.2.4) hold for some n.

i.e. $x_n \leq x_{n+1}$ and $y_n \geq y_{n+1}$

We shall prove that (3.2.4) is true for n+1.Now $x_n \leq x_{n+1}$and $y_n \geq y_{n+1}$ then by mixed monotone property of F we have

x_{n+2}= F$(x_{n+1}, y_{n+1}) \geq$ F$(x_n, y_{n+1}) \geq$ F$(x_n, y_n) = x_{n+1}$
and

$$y_{n+2} = F(y_{n+1}, x_{n+1}) \leq F(y_n, x_{n+1}) \leq F(y_n, x_n) = y_{n+1}$$

Thus by mathematical induction we conclude that (3.2.4) holds for all n ≥ 0
So,

$$x_0 \leq x_1 \leq x_2 \leq ....x_n \leq x_{n+1} \leq ....$$

and

$$y_0 \geq y_1 \geq y_2 \geq ....y_n \geq y_{n+1} \geq ....$$

Since, $x_{n-1} \leq x_n$ and $y_{n-1} \geq y_n$,
Put $x = x_n$, $y = y_n, u = x_{n-1}, v = y_{n-1}$ in (3.2.1)
We have
$M(F(x_n, y_n), F(x_{n-1}, y_{n-1}), kt) \geq \varphi(M(x_n, x_{n-1}, t))$
$M(x_{n+1}, x_n, kt) \geq M(x_n, x_{n-1}, t)$

$$\Rightarrow M(x_{n+1}, x_n, t) \geq M\left(x_n, x_{n-1}, \frac{t}{k^1}\right) \geq M\left(x_{n-1}, x_{n-2}, \frac{t}{k^2}\right) \geq M\left(x_{n-2}, x_{n-3}, \frac{t}{k^3}\right) \vdots \vdots \geq M(x_{n-(n-1)}, x_{n-n,} \frac{t}{k^n}) \geq M(x_1, x_{0,} \frac{t}{k^n})$$

$\Rightarrow \lim_{n\to\infty} M(x_{n+1}, x_n, t) = 1.$
We now verify that $\{x_n\}$ is a Cauchy sequence.
So, M $(x_n, x_{n+p,}$t) ≥ $M(x_n, x_{n+1}, t) * \cdots * M(x_{n+p-1}, x_{n+p}, t)$
$\Rightarrow \lim_{n\to\infty} M(x_n, x_{n+p}, t) \geq 1 * \cdots * 1 = 1$
$\Rightarrow \lim_{n\to\infty} M(x_n, x_{n+p}, t) = 1.$
$\Rightarrow \{x_n\}$ is a Cauchy sequence.
Again, Since $y_{n-1} \geq y_n$ and $x_{n-1} \leq x_n$
put $x = y_{n-1}$, $y = x_{n-1}, u = y_n, v = x_n$ in (3.2.1) we have
$M(F(y_{n-1}, x_{n-1}), F(y_n, x_n), kt) \geq \varphi(M(y_{n-1}, y_n, t))$
M $(y_n, y_{n+1}, kt) \geq M(y_{n-1}, y_n, t)$

$$\Rightarrow M(y_n, y_{n+1}, t) \geq M\left(y_{n-1}, y_n, \frac{t}{k^1}\right) \geq M\left(y_{n-2}, y_{n-1}, \frac{t}{k^2}\right) \geq M\left(y_{n-3}, y_{n-2}, \frac{t}{k^3}\right) \vdots \vdots \geq M(y_{n-n,} y_{n-(n-1)} \frac{t}{k^n}) \geq M(y_{0,} y_1 \frac{t}{k^n})$$

$\Rightarrow \lim_{n\to\infty} M(y_n, y_{n+1}, t) = 1.$
We now verify that $\{y_n\}$ is a Cauchy sequence.
So, M $(y_n, y_{n+p,}$t) ≥ $M(y_n, y_{n+1}, t) * \cdots * M(y_{n+p-1}, y_{n+p}, t)$
$\Rightarrow \lim_{n\to\infty} M(y_n, y_{n+p}, t) \geq 1 * \cdots * 1 = 1$
$\Rightarrow \lim_{n\to\infty} M(y_n, y_{n+p}, t) = 1.$
$\Rightarrow \{y_n\}$ is a Cauchy sequence.
Now it is clear that $\{x_n\}$ and $\{y_n\}$ are Cauchy sequence in X.
Since X is a complete fuzzy metric space, there exist x, y ∈ X such that
$\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$
Now suppose that assumption (a) holds.
Thus by taking limit as $n \to \infty$ in (3.2.3), we get
$x = \lim_{n\to\infty} x_n = \lim_{n\to\infty} F(x_{n-1,} y_{n-1}) = F(\lim_{n\to\infty} x_{n-1}, \lim_{n\to\infty} y_{n-1}) = F(x, y)$
and
$y = \lim_{n\to\infty} y_n = \lim_{n\to\infty} F(y_{n-1,} x_{n-1}) = F(\lim_{n\to\infty} y_{n-1}, \lim_{n\to\infty} x_{n-1}) = F(y, x).$
Hence x=F(x , y) and y=F(y ,x).
Finally suppose that (b) holds. If $\{x_n\}$ is non decreasing sequence $\{x_n\} \to x$ and as $\{y_n\}$ is non -decreasing sequence and $\{y_n\} \to y$,by assumption (b) we have $x_n \leq x$ and $y \leq y_n$ for all n.
We have $M(F(x_n, y_n), F(x, y), kt) \geq \varphi(M(x_n, x, t))$ (3.2.5)
Taking n→ ∞ in (3.2.5),
we get $M(x, F(x, y), kt) = 1 \Rightarrow F(x, y) = x.$
Similarly we can show that $F(y, x) = y.$
Hence we conclude that F has a coupled fixed point.

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