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FINITE SUMMATION FORMULAE FOR MULTIVARIABLE I-FUNCTION

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Abstract

In the present paper, on attempt has been made to derive finite summation formulae formulae for the multivariable I-function introduced by C.K. Sharma [8] since the multivariable I-Function Includes a large number of special function one and more variable as its particular cases, the result established here serve as key formulae giving as a large number of new and interesting result by specializing the parameter involved.

Introduction and Notation

For the multivariable I-Function which was introduced by Sharma and Panday ([6], [7]) which in an extension of the multivariable H-function. This multivariable I-Function includes I-function, fox's H-Function and Meiger's G-Function of one and two variable, the generalized Lauricella function defined by Shrivastava and Daoust [4]. Appeal function the Whittaker function therefore the result established in this paper are of a general character and hence encompass several cases of interest.

The object of this paper is to establish four finite summation formulae for the multivariable I-Function these formulae will yeilda number of new and know results including the result of Gupta and Garg [2], [3].

The multivariable I-Function defined by Sharma and Panday. Since only the Parameter which subscript 1 in the definition of the multivariable I-Function (8) undergo changes in our summation formulae that following, to simplify notation problems, we specify and these parameter in them. Thus I $\left[(a_1 - r; h, k), (b_1 - r, ; \beta_h, \beta_k)\ \right]$ would represent the multivariable I-Function defined Sharma. But having a_1 replaced by a_1 –r, $\alpha_1^{(r)}$ replaced by $h^{(i)}$ (i=1,2, ...r), b_1 replaced by b_1 –r, $\beta_h^{(i)}(i = 1,2,.........-r)$ the last of the parameters remaining unchanged and so on we shall give below three-term contiguous relation for the multivariable I-Function and use them later on.

(i) $$\left[\sqrt{(1-a_1} + r + \frac{b_1}{\beta})\right]^{-1} I\ [(a_1 + 1 - r;\ h^1 \quad , h^{(r)}\ , b_1 + 1, -i - \beta h^1 \quad \beta h^{(r)}\ ]$$

=

$$\beta\ [\sqrt{(1-a_1} + r + \frac{b_1}{\beta})]\ I\ [(a_1 - r, h^1 ....., \ h^{(r)}), (b_{1,}\ \beta_h{}^1 ....\beta_h{}^{(r)})].......(1.1)$$

(ii) $$\beta(-1)^r [\left| (b_1 + r - a_1\beta)\right]^{-1}\ I[(a_1, h^1 ....h^{(r)}), (b_1 + r - 1;\ \beta h^1 ....\beta h^r)\ \|$$

=

$$(-1)^n [(b_1 + r - aj\beta)]^{-1}\ I\ [(a_1 + 1jh^1, ...h^{(r)}], (b_1 + n;\ \beta h^1 ....\beta h^{(r)})]$$

$$-\ [(b_1 - a_1\beta)]^{-1}\ I[a_1 + 1; h^1 ....h^{(r)}, (b_1;\ \beta h^1, ...\beta h^{(r)})]$$

$$+ (1)^r [(b_1 + r - 1 - a_1\beta)]^{-1} [(a_1 + 1; h^1 ....h^{(r)}), b_1 + r - 1,\ \beta h^1 ;.......\beta^{h(r)}]$$

(iii) $|r-b_1 1-a_1+r|\ I\ [(a_1-r;\ h^1,..........h^{(r)}),\ b_1-r,\beta h^1.......ph^{(r)}]$

$= I\ [(a_1-r;.......h^{(1)},h^{(r)}),b_1-(r+1);\ \beta h^1,......\beta h^{(r)}]$

$-\beta I[(aj-r-1;\ h_1^{\ 3},.......h^{(r)}]\ (b_1-r;\ \beta h^1........\beta_h^{\ (r)}]----(1.3)$

(iv) $(b_1-a_1+1)\ I[(aj;h^1,.....h^{(r)}),(b_1,\beta h^1........\beta h^{(r)})\ ]$

(v) =

$I[a_1-1,h^1,...........h^{(r)}),(b_1;\beta h^1,.............\beta h^{(r)})]-I\ [(aj;h^1,.......h^{(r)}],(b_1+1;\beta h^1.....\beta h^r)]$

The contigeous relation (1.1) , (1.2) and (1.3), (1,4) can be developed on lines similar to those given by Buchman and Gupta {1}.(1).

Finite summation formulae

The finite summation formulae to be established are :-

(i).

$$\sum_{r=1}^{n}[(1-a_1+r+\frac{b_1}{\beta})]^{-1}\ I[(a_1+1-r;\ h_1^{\ 1}......h^{(r)}),(b_1+1;\beta h^1;........\beta h^{(r)})]$$

$$=\beta[\ (1-a_1+n+\frac{b_1}{\beta})]^{-1}\ I\ [(a_1-n;h^1,......h^{(r)}),(b_1;\ \beta h^1.......;\beta h^{(r)})]$$

$$-\beta\ [(1-a_1+\frac{b_1}{\beta})]^{-1}\ I\ [(a_1\ ;\ h_1......h^{(r)}),\ (b_1\ ;\ \beta h^1,......\beta h^{\ (r)})\ ]$$

.................(2.1)

(ii).

$$\beta\sum_{r=1}^{n}(1)^r[\ (b_1+r-a_1\beta\ )\ ]^{-1}\ I\ [\ (a_1;\ h^{(1)},.......h^{(r)}),\ (b_1+r-1;\ \ \beta h^1.....\beta h^{(r)})]$$

$$=(-1)^n\ [\ (b_1+r-aj\beta)\ ]^{-1}\ I[\ (a_1+1;h^1;......h^{(r)}),\ (b_1+n;\beta h^1......\beta h^{(r)})]$$

$$-[\ (b_1-a_1\beta)]^{-1}\ I\,(a_1+1;h^1,......h^{(r)},(b_1;\ \beta h^1,\ \ \beta h^{(r)})].......(2.2)$$

(iii)

$$\sum_{r=1}^{n}|r-b_1\ \ 1-a_1+r\ \ |\ \beta^{r-1}\ I[\ (a_1-r,h^1,.......h^r\ ;\ b_1-r;\ \ \beta h^1,.....\beta h^{(r)}]$$

$$= I\ [\ (\ a_1-1;\ h^1,.......h^{(r)}),\ (b_1;\beta h^1,..........\beta h^{(r)})\ ]$$

$$-\beta\ \ I\ [\ (a_1-n-1);\ h^1,.......h^{(r)});\ (b_1-n;\beta h^1..........\beta h^{(r)})]$$

(iv)

$$\sum_{r=1}^{n}(-1)^{n}\ (n)\ i\ [(a_1-n+r;h^1..........,h^{(r)}),(b_1+r;\ \beta h^1,....\beta h^{(r)}$$

$$=(b_1\ -\ a_1\ +\ z)_n\ I[(a_1;h^1,........,h^{(r)}),\ (b_1;\beta h^1....\beta h^{(r)})].........(2.4)$$

provided that the series involved in all the above formulae is absolutely convergent .

Proof : To prove (2.1) putting r=1, 2 .... n in (1.1) in sucession and after taking the sum, we see that in the resulting reins on the right hand side, after that terms cancel art and we arrive at the required result (2.1).

Similarly, (2.2) and (2.3) can be established by using the results (1.2) and (1.3) respecting in place of (1.1) (multiplying by quantities 1, β β^2 , β^{n-1} respectively only for (2.3).

To prove (2.4), if we iterate by expanding each term on the right hand side of (1.4) by the use of this and we do, not write the repeat parameters h^1 .... $h^{(r)}$ again and again containing this process of iteration, we finally arrive at the required result (2.4).

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