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A STUDY ON (E,Q)(E, Q) PRODUCT SUMMABILITY OF FOURIER SERIES AND ITS CONJUGATE SERIES

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Abstract

In this paper, introduce the concept of (E,q) (E,q) product operators and establishes two new theorems on (E,q)(E,q) product Summability of Fourier series and its conjugate series. The results obtained in the paper further extend several known result on linear operators.

Keyword: (E,q) Summability, (E,q) (E,q) Summability.

1. INTRODUCTION

In this field of Summability of Fourier series & its allied series, the product Summability (E,q)(X),(X)(E,q) or |E,q| have be studied by a number of researchers like, Jadia[2], khare[3], Mittal and kumar [5], Pandey[6], singh[7], singh and singh[8] have studied (N,P_n), (N,P,q), almost (N,P,q) and matrix series using different condition, matrix method includes (N,P_n) and (N,p,q) method of Summability as special cases. Thereafter, Chandra & Dixit [9] studied |B| and |E,q| and Summability of fourier series and its allied series. But nothing seems to have been so far to study (E,q)(E,q) product Summability of fourier series and its conjugate series. Therefore, in this paper, two theorems on (E,q)(E,q) Summability of fourier series and its conjugate series under a general condition have been proved.

2. DEFINATION AND NOTATION

Let f(x) be a 2π- periodic function and Lebesgue integrable over (-π,π). The Fourier series of f(x) is given by

$$f(x)\sim \frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx)\equiv\sum_{n=1}^{\infty}A_n(x)$$
(2.1)

The conjugate series of Fourier series is given b $\sum_{n=1}^{\infty}(b_n\cos nx-a_n\sin nx)\equiv\sum_{n=1}^{\infty}B_n(\mathbf{x})$
(2.2)

We shall use the following notations:

$\Phi(t)$ = f(x+t)+f(x-t)-2S

$\psi(t)=\frac{1}{2}\{f(x+t) - f(x-t)\}$

$$K_n(t)=\frac{1}{2\pi(1+q)^n}\sum_{k=0}^{n}\left[\frac{1}{(1+q)^k}\binom{n}{k}q^{n-k}\left\{\sum_{v=0}^{k}\binom{k}{v}q^{k-v}\frac{\sin\left(v+\frac{1}{2}\right)t}{\sin\frac{t}{2}}\right\}\right]$$

$$\widetilde{K_n}(t)=\frac{1}{2\pi(1+q)^n}\sum_{k=0}^{n}\left[\frac{1}{(1+q)^k}\binom{n}{k}q^{n-k}\left\{\sum_{v=0}^{k}\binom{k}{v}q^{k-v}\frac{\cos\left(v+\frac{1}{2}\right)t}{\sin\frac{t}{2}}\right\}\right]$$

And $\tau=\left[\frac{1}{t}\right]$, where τ denotes the greatest integer not greater than $\frac{1}{t}$

Let $\sum_{n=0}^{\infty}u_n$ be a given infinite series with sequence of its n^{th} partial sum of $\{S_n\}$. The (E,q) transform is defined as the n^{th} partial sum of (E,q) Summability and is given by

$$E_k^q=(E,q)=\frac{1}{(1+q)^k}\sum_{v=0}^{k}\binom{k}{v}q^{k-v}\ S_v \text{ as } n\to\infty \tag{2.3}$$

Then the infinite series $\sum_{n=0}^{\infty}u_n$ is summable to the definite number s by (E,q)(E,q) Summability method.

If,

$$t_n^{(E,q)(E,q)}=E_n^qE_k^q=\frac{1}{(1+q)^n}\sum_{k=0}^{n}\binom{n}{k}q^{n-k}\ E_k^q\to s \text{ as } n\to\infty$$
(2.4)

3. MAIN THEOREMS

We prove the following theorems,

3.1 Theorem. Let $\{p_n\}$ be a positive, monotonic, non-increasing sequence of real constants such that

$$p_n = \sum_{v=0}^{n} p_v \to \infty \text{ as } n \to \infty$$

if,

$$\emptyset(t) = \int_0^t |\emptyset(u)| du = o\left[\frac{t}{\alpha(\frac{1}{t})}\right], \text{as } t \to +0 \qquad (3.1)$$

Where, α(t) is positive, monotonic and non-increasing function of t and $\alpha(n)$ as $n \to \infty$

Then the Fourier series (2.1) is summable (E,q)(E,q) to f(x).

3.2 Theorem. Let $\{p_n\}$ be a positive, monotonic, non-increasing sequence of real constants such that

$$p_n = \sum_{v=0}^{n} p_v \to \infty \text{ as } n \to \infty$$

If,

$$\psi(t) = \int_0^t |\psi(u)| du = o\left[\frac{t}{\alpha(\frac{1}{t})}\right], \text{as } t \to +0 \qquad (3.2)$$

where α(t) is a positive, monotonic and non-increasing function of t, then the conjugate Fourier series (2.2) is summable to (E,q)(E,q) to

$$\tilde{f}(x) = \frac{-1}{2\pi}\int_0^{2\pi} \psi(t) \cot\left(\frac{t}{2}\right) dt$$

at any point where this point exists.

4. LEMMAS

Lemma 1. $|k_n(t)| = O(n), \quad \text{for } 0 \le t \le \frac{1}{n}; \sin nt \le n \sin t; |\cos nt \le 1|$

Proof:

$$|k_n(t)| \le \frac{1}{2\pi(1+q)^n}\left|\sum_{k=0}^{n}\left[\frac{1}{(1+q)^k}\binom{n}{k}q^{n-k}\sum_{v=0}^{k}\binom{k}{v}q^{k-v}\frac{\sin\left(v+\frac{1}{2}\right)t}{\sin\frac{t}{2}}\right]\right|$$

$$\le \frac{1}{2\pi.(1+q)^n}\left|\sum_{k=0}^{n}\left[\frac{1}{(1+q)^k}\binom{n}{k}q^{n-k}\sum_{v=0}^{k}\binom{k}{v}q^{k-v}\frac{(2v+1)\sin\frac{t}{2}}{\sin\frac{t}{2}}\right]\right|$$

$$\le \frac{1}{2\pi.(1+q)^n}\left|\sum_{k=0}^{n}\left[\frac{1}{(1+q)^k}\binom{n}{k}q^{n-k}(2k+1)\sum_{v=0}^{k}\binom{k}{v}q^{k-v}\right]\right|$$

$$= \frac{1}{2\pi.(1+q)^n}\sum_{k=0}^{n}\binom{n}{k}q^{n-k}(2k+1)$$

$$= \frac{1}{2\pi.(1+q)^n}(2n+1)\sum_{k=0}^{n}\binom{n}{k}q^{n-k}$$

$$= \frac{2n+1}{2\pi.(1+q)^n}$$

$$= O(n)$$

Lemma 2 $|k_n(t)| = o\left(\frac{1}{tn}\right), \text{for } \frac{1}{n} \le t \le \pi; \sin(t/2) \ge t/2 \text{ and } \sin nt \le 1$

$$|\mathbf{k_n(t)}| \le \frac{\mathbf{1}}{2\pi.(1+q)^n}\left|\sum_{\mathbf{k=0}}^{\mathbf{n}}\left[\frac{\mathbf{1}}{(\mathbf{1+q})^{\mathbf{k}}}\binom{n}{k}q^{n-k}\sum_{\mathbf{v=0}}^{\mathbf{k}}\binom{\mathbf{k}}{\mathbf{v}}\mathbf{q^{k-v}}\frac{\mathbf{\sin\left(v+\frac{1}{2}\right)t}}{\mathbf{\sin\frac{t}{2}}}\right]\right|$$

$$\le \frac{\mathbf{1}}{2\pi.(1+q)^n}\left|\sum_{\mathbf{k=0}}^{\mathbf{n}}\left[\frac{\mathbf{1}}{(\mathbf{1+q})^{\mathbf{k}}}\binom{n}{k}q^{n-k}\sum_{\mathbf{v=0}}^{\mathbf{k}}\binom{\mathbf{k}}{\mathbf{v}}\mathbf{q^{k-v}}.\frac{\mathbf{1}}{\mathbf{t/2}}\right]\right|$$

$$= \frac{\mathbf{1}}{\pi t.(1+q)^n}\left[\sum_{\mathbf{k=0}}^{\mathbf{n}}\binom{n}{k}q^{n-k}\sum_{\mathbf{v=0}}^{\mathbf{k}}\binom{\mathbf{k}}{\mathbf{v}}\mathbf{q^{k-v}}\right]$$

$$= \frac{\mathbf{1}}{\pi t.(1+q)^n}$$

$$= O\left(\frac{1}{tn}\right)$$

Lemma 3. $\widetilde{k_n}(t) = O\left(\frac{1}{tn}\right), \text{for } 0 \le t \le \frac{1}{n}; \sin(t/2) \ge t/2; |\cos nt| \le 1$

$$|\widetilde{k_n}(t)| \le \frac{1}{2\pi.(1+q)^n}\left|\sum_{k=0}^{n}\left[\frac{1}{(1+q)^k}\binom{n}{k}q^{n-k}\sum_{v=0}^{k}\binom{k}{v}q^{k-v}\frac{\cos\left(v+\frac{1}{2}\right)t}{\sin\frac{t}{2}}\right]\right|$$

$$\le \frac{1}{2\pi.(1+q)^n}\sum_{k=0}^{n}\left[\frac{1}{(1+q)^k}\binom{n}{k}q^{n-k}\sum_{v=0}^{k}\binom{k}{v}q^{k-v}\left|\frac{\cos\left(v+\frac{1}{2}\right)t}{\sin t/2}\right|\right]$$

$$= \frac{1}{\pi t.(1+q)^n}\sum_{k=0}^{n}\left[\frac{1}{(1+q)^k}\binom{n}{k}q^{n-k}\sum_{v=0}^{k}\binom{k}{v}q^{k-v}\right]$$

$$= \frac{1}{\pi t.(1+q)^n}\left[\sum_{k=0}^{n}\binom{n}{k}q^{n-k}\right]$$

$$= \frac{1}{\pi t.(1+q)^n}$$

$$= O\left(\frac{1}{tn}\right)$$

Lemmas 4. $|\widetilde{k_n}(t)| = O\left(\frac{1}{tn}\right)$, for $\frac{1}{n} \le t \le \pi, \sin({}^{t}/_{2}) \ge {}^{t}/_{2}$

Proof:-

$$|\widetilde{k_n}(t)| \le$$

$$\frac{1}{2\pi.(1+q)^n}\left|\sum_{k=0}^{n}\left[\frac{1}{(1+q)^k}\binom{n}{k}q^{n-k}\sum_{v=0}^{k}\binom{k}{v}q^{k-v}.\frac{\cos\left(v+\frac{1}{2}\right)t}{\sin\frac{t}{2}}\right]\right|$$

$$\le \frac{1}{\pi t.(1+q)^n}\left|\sum_{k=0}^{n}\left[\frac{1}{(1+q)^k}\binom{n}{k}q^{n-k}\mathrm{Re}\left\{\sum_{v=0}^{k}\binom{k}{v}q^{k-v}e^{i\left(v+\frac{1}{2}\right)t}\right\}\right]\right|$$

$$\le \frac{1}{\pi t.(1+q)^n}\left|\sum_{k=0}^{n}\left[\frac{1}{(1+q)^k}\binom{n}{k}q^{n-k}\mathrm{Re}\left\{\sum_{v=0}^{k}\binom{k}{v}q^{k-v}e^{ivt}\right\}\right]\right|\left|e^{it/2}\right|$$

$$\le \frac{1}{\pi t.(1+q)^n}\left|\sum_{k=0}^{n}\left[\frac{1}{(1+q)^k}\binom{n}{k}q^{n-k}\mathrm{Re}\left\{\sum_{v=0}^{k}\binom{k}{v}q^{k-v}e^{ivt}\right\}\right]\right|$$

$$\le \frac{1}{\pi t.(1+q)^n}\left|\sum_{k=0}^{\tau-1}\left[\frac{1}{(1+q)^k}\binom{n}{k}q^{n-k}\mathrm{Re}\left\{\sum_{v=0}^{k}\binom{k}{v}q^{k-v}e^{ivt}\right\}\right]\right|$$

$+$

$$\frac{1}{\pi t.(1+q)^n}\left|\sum_{k=\tau}^{n}\left[\frac{1}{(1+q)^k}\binom{n}{k}q^{n-k}\mathrm{Re}\left\{\sum_{v=0}^{k}\binom{k}{v}q^{k-v}e^{ivt}\right\}\right]\right|$$

$$= k_1 + k_2$$

Now,

$|k_1|$

$$\le \frac{1}{\pi t.(1+q)^n}\left|\sum_{k=0}^{\tau-1}\left[\frac{1}{(1+q)^k}\binom{n}{k}q^{n-k}\mathrm{Re}\left\{\sum_{v=0}^{k}\binom{k}{v}q^{k-v}e^{ivt}\right\}\right]\right|$$

$$\le \frac{1}{\pi t.(1+q)^n}\left|\sum_{k=0}^{\tau-1}\left[\frac{1}{(1+q)^k}\binom{n}{k}q^{n-k}\sum_{v=0}^{k}\binom{k}{v}q^{k-v}\right]\right|\left|e^{ivt}\right|$$

$$\le \frac{1}{\pi t.(1+q)^n}\left|\sum_{k=0}^{\tau-1}\frac{1}{(1+q^k)}\binom{n}{k}q^{n-k}\sum_{v=0}^{k}\binom{k}{v}q^{k-v}\right|$$

$$\le \frac{1}{\pi t.(1+q)^n}\sum_{k=0}^{\tau-1}\binom{n}{k}q^{n-k}$$

$$= \frac{1}{\pi t.(1+q)^n}$$

$$= O\left(\frac{1}{tn}\right)$$

And

$|k_2|$

$$\le \frac{1}{\pi t.(1+q)^n}\left|\sum_{k=\tau}^{n}\left[\frac{1}{(1+q)^k}\binom{n}{k}q^{n-k}\mathrm{Re}\left\{\sum_{v=0}^{k}\binom{k}{v}q^{k-v}e^{ivt}\right\}\right]\right|$$

$$\le \frac{1}{\pi t.(1+q)^n}\sum_{k=\tau}^{n}\frac{1}{(1+q)^k}\binom{n}{k}q^{n-k}\max_{0\le m\le k}\left|\sum_{v=0}^{k}\binom{k}{v}q^{k-v}e^{ivt}\right|$$

$$\le \frac{1}{\pi t.(1+q)^n}(1+q)^\tau\sum_{k=\tau}^{n}\frac{1}{(1+q)^k}\binom{n}{k}q^{n-k}$$

$$= \frac{1}{\pi t.(1+q)^n}\sum_{k=\tau}^{n}\binom{n}{k}q^{n-k}$$

$$\frac{1}{\pi t.(1+q)^n}$$

$$= O\left(\frac{1}{tn}\right)$$

5. PROOF OF MAIN THEOREMS

5.1 Proof of theorem

Following Titchmash [8] and using Riemann-Lebesgue theorem, $S_n(f;x)$ of the series (2.1) is given by

$$S_n(f;x) - f(x) = \frac{1}{2\pi}\int_0^\pi \emptyset(t)\frac{\sin\left(n+\frac{1}{2}\right)t}{\sin\frac{t}{2}}dt$$

Therefore using (2.1), the (E,q), transform E_k^q of $S_n(f;x)$ is given by

$$E_k^q - f(x)$$

$$= \frac{1}{2\pi(1+q)^n}\int_0^\pi \emptyset(t)\left\{\sum_{v=0}^{k}\binom{k}{v}q^{k-v}\frac{\sin\left(k+\frac{1}{2}\right)t}{\sin\frac{t}{2}}\right\}dt$$

Now denoting (E,q)(E,q) transform of $S_n(f;x)$ by $E_n^qE_k^q$ we write

$$E_n^q E_k^q - f(x)$$

$$= \frac{1}{2\pi(1+q)^n}\sum_{k=0}^{n}\left[\frac{1}{(1+q)^k}\binom{n}{k}q^{n-k}\int_0^\pi \phi(t)\left\{\sum_{v=0}^{k}\binom{k}{v}q^{k-v}\frac{\sin\left(k+\frac{1}{2}\right)t}{\sin\frac{t}{2}}\right\}dt\right]$$

$$= \int_0^\pi \phi(t)k_n(t)dt \qquad (5.1)$$

Where,

$k_n(t)$

$$= \frac{1}{2\pi(1+q)^n}\sum_{k=0}^{n}\left[\frac{1}{(1+q)^k}\binom{n}{k}q^{n-k}\left\{\sum_{v=0}^{k}\binom{k}{v}q^{k-v}\frac{\sin\left(k+\frac{1}{2}\right)t}{\sin\frac{t}{2}}\right\}\right]$$

We have to show that, under the hypothesis of theorem

$$\int_0^\pi \phi(t)k_n(t)dt = o(1), \text{as } n\to\infty$$

For $0<\delta<\pi$, We have

$$\int_0^\pi \phi(t)k_n(t)dt = \left[\int_0^{1/n}\phi(t) + \int_{1/n}^{\delta}\phi(t) + \int_\delta^\pi \phi(t)\right]k_n(t)dt$$

$$= I_1 + I_2 + I_3 \text{ (Say)} \qquad (5.2)$$

We consider,

$$|I_1| \le \int_0^{1/n}|\phi(t)||k_n(t)|dt$$

$$= O(n)\left[\int_0^{1/n}|\phi(t)|dt\right] \text{ by lemma 1}$$

$$= O(n)\left[o\left\{\frac{1}{n\alpha(n)}\right\}\right] \text{ by (3.1)}$$

$$= o\left\{\frac{1}{\alpha(n)}\right\}$$

$$= o(1), \text{as } n\to\infty \qquad (5.3)$$

Now,

$$|I_2| \le \int_{1/n}^{\delta}|\phi(t)||k_n(t)|dt$$

$$= O\left[\int_{1/n}^{\delta}|\phi(t)|\left(\frac{1}{tn}\right)dt\right] \text{ by lemma 2}$$

$$= O\left(\frac{1}{n}\right)\left[\left\{\frac{1}{t}\phi(t)\right\}_{1/n}^{\delta} + \int_{1/n}^{\delta}\frac{1}{t^2}\phi(t)\,dt\right]$$

$$= O\left(\frac{1}{n}\right)\left[o\left\{\frac{1}{\alpha\left(\frac{1}{t}\right)}\right\}_{1/n}^{\delta} + \int_{1/n}^{\delta} o\left(\frac{1}{t\alpha\left(\frac{1}{t}\right)}\right)dt\right] \text{ by (3.1)}$$

Putting $\frac{1}{t}=u$ in second term,

$$= O\left(\frac{1}{n}\right)\left[o\left\{\frac{1}{\alpha(n)}\right\} + \int_{1/\delta}^{n} o\left(\frac{1}{u\alpha(u)}\right)du\right]$$

$$= o\left\{\frac{1}{\alpha(n)}\right\} + o\left\{\frac{1}{n\alpha(n)}\right\}\int_{1/\delta}^{n}1.du$$

Using second mean value theorem for the integral in the second term as $\alpha(n)$ is monotonic

$$= o(1) + o(1) \text{ as}, n\to\infty$$

$$= o(1), \text{as } n\to\infty \qquad (5.4)$$

By Rieman-Lebesgue theorem and by regularity condition of the method of Summabilty,

$$|I_3| \le \int_\delta^\pi |\phi(t)||k_n(t)|dt$$

$$= o(1), \text{as } n\to\infty \qquad (5.5)$$

Combining (5.3), (5.4) and (5.5) we have

$$E_n^q E_k^q - f(x) = o(1), \text{as } n\to\infty$$

This completes the proof of theorem 1.

5.2 Proof of Theorem. Let $\tilde{s_n}(f;x)$ denotes the partial sum of series (2.2).

Then following Lal[4] and using Riemann-Lebesgue Theorem ,$\widetilde{s_n}(f;x)$ of series (2.2) is given by

$$\tilde{s_n}(f;x) - \tilde{f}(x) = \frac{1}{2\pi}\int_0^\pi \phi(t)\frac{\cos\left(n+\frac{1}{2}\right)t}{\sin\frac{t}{2}}dt$$

Therefore using (2.2), the (E,q) transform E_k^q of $\tilde{s_n}(f;x)$ is given by

$$\tilde{E}_k^q - \tilde{f}(x)$$

$$= \frac{1}{2\pi(1+q)^k}\int_0^\pi \psi(t)\left\{\sum_{v=0}^{k}\binom{k}{v}q^{k-v}\frac{\cos\left(k+\frac{1}{2}\right)t}{\sin\frac{t}{2}}\right\}dt$$

Now denoting $\overline{(E,q)(E,q)}$ transform of $\tilde{s_n}(f;x)$ by $\overline{E_n^q E_k^q}$ we write

$\overline{E_n^q E_k^q} - \tilde{f}(x)$

$$= \frac{1}{2\pi(1+q)^n}\sum_{k=0}^{n}\left[\frac{1}{(1+q)^k}\binom{n}{k}q^{n-k}\int_0^\pi \psi(t)\left\{\sum_{v=0}^{k}\binom{k}{v}q^{k-v}\frac{\cos\left(k+\frac{1}{2}\right)t}{\sin\frac{t}{2}}\right\}dt\right]$$

$$= \int_0^\pi \psi(t)\widetilde{k_n}(t)dt \tag{5.6}$$

In order to prove the Theorem, we have to show that, under the hypothesis of theorem

$$\int_0^\pi \psi(t)\widetilde{k_n}(t)dt = o(1) \quad \text{as } n\to\infty$$

For $0< \delta < \pi$, we have

$$\int_0^\pi \psi(t)\widetilde{k_n}(t)dt = \left[\int_0^{1/n}\psi(t) + \int_{1/n}^{\delta}\psi(t) + \int_\delta^\pi \psi(t)\right]\widetilde{k_n}(t)dt$$

$$= J_1 + J_2 + J_3 \quad \text{(Say)} \tag{5.7}$$

We consider,

$$|J_1 \le |\int_0^{1/n} |\psi(t)|\,|\widetilde{k_n(t)}|\,dt$$

$$= O\left[\int_0^{1/n}\frac{1}{tn}|\psi(t)|dt\right] \quad \text{by lemma 3}$$

$$= O\left(\frac{1}{n}\right)\left[\int_0^{1/n}\frac{1}{t}|\psi(t)|\,dt\right]$$

$$= O(n)\left(\frac{1}{n}\right)\left[o\left\{\frac{1}{n\alpha(n)}\right\}\right] \text{ By 3.1}$$

$$= o\left\{\frac{1}{\alpha(n)}\right\}$$

$$= o(1), \text{ as } n\to\infty \tag{5.8}$$

Now, $|J_2| \le \int_{1/n}^{\delta}|\psi(t)|\,|\widetilde{k_n}(t)|dt$

$$= O\left[\int_{1/n}^{\delta}\frac{1}{tn}|\psi(t)|dt\right] \text{ by lemma 4}$$

$$= O\frac{(1)}{n}\left[\int_{1/n}^{\delta}\frac{1}{t}|\psi(t)|dt\right]$$

$$= O\left(\frac{1}{n}\right)\left[\left\{\frac{1}{t}\psi(t)\right\}_{1/n}^{\delta} + \int_{1/n}^{\delta}\frac{1}{t^2}\psi(t)dt\right]$$

$$= O\left(\frac{1}{n}\right)\left[o\left\{\frac{1}{\alpha\left(\frac{1}{t}\right)}\right\}_{1/n}^{\delta} + \int_{1/n}^{\delta} o\left(\frac{1}{t\alpha\left(\frac{1}{t}\right)}\right)dt\right] \text{ By (3.3)}$$

Putting $\frac{1}{t} = u$, in second term,

$$= O\left(\frac{1}{n}\right)\left[o\left\{\frac{1}{\alpha(n)}\right\} + \int_{1/\delta}^{n} o\left(\frac{1}{u\alpha(u)}\right)du\right]$$

$$= o\left\{\frac{1}{\alpha(n)}\right\} + o\left\{\frac{1}{n\alpha(n)}\right\}\int_{1/\delta}^{n} 1.\,du$$

Using second -mean value theorem for the integral in the second term as $\alpha(n)$ is monotonic

$$= o(1) + o(1), \text{ as } n\to\infty$$

$$= o(1), \text{ as } n\to\infty \tag{5.9}$$

By Riemann – Lebesgue theorem and by regularity condition of the method of Summability

$$|J_3| \le \int_\delta^\pi |\psi(t)|\,|\widetilde{k_n}(t)|\,dt$$

$$= o(1), \text{ as } n\to\infty \tag{5.10}$$

Combining (5.8) , (5.9) and (5.10) we have,

$$\overline{E_n^q E_k^q} - \tilde{f}(x) = o(1), \text{ as } n\to\infty$$

This completes the proof of theorem 2.

CONCLUSION

In the field of Summability theory, various results pertaining (E,q) and (E,q), (E,q)X and X(E,q) Summability of Fourier series as well as its allied series have been reviewed.

In future, the present work can be extended to establish new results under certain conditions.

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