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A FIXED POINT THEOREM IN NON ARCHIMEDEAN INTUITIONISTIC FUZZY 3 METRIC SPACES

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Abstract

In this paper,We prove a fixed point theorem for a commuting maps and our result generalize and extend some recent results for non Archimedean intuitionistic fuzzy 3 metric spaces.
Key words: - non Archimedean fuzzy metric space, fixed point, commuting maps.

1. Introduction

Zadeh [24] introduced the concept of fuzzy metric space in 1965.Many authors have developed the theory of fuzzy sets and applications .Deng [23], Erceg[12],Kaleva and Seikkala [16] and many authors gave the concept of fuzzy metric space in different ways.Grabiec[14] introduced the fixed point theory in fuzzy metric space.Atanassov gave the concept of intuitionistic fuzzy metric sets as a generalization of fuzzy sets.The concept of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms is given by Park[10] in 2004.Then using the idea of intuitionistic fuzzy metric sets Alaca et al. defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [9] in 2006.After that Jungck'[8] common fixed point theorem of intuitionistic fuzzy metric space for commuting mappings.The concept of fuzzy 2-metric space is given by Sushil Sharma[21] in 2002 and he also proved common fixed point theorem in fuzzy 2-metric space.We introduce the concept of non Archimedean intuitionistic fuzzy 3 metric spaces by using the concept of Archimedean fuzzy metric space by Dorel Mihet[4],Sushil Sharma[21] and Renu Chugh and Sumitra[17] .

2. Preliminaries

Definition 2.1[15] A binary operation$*$: $[0,1] \times [0,1] \to [0,1]$ is continuous t-norm if $*$ satisfies the following conditions:

(i) $*$ is commutative and associative;
(ii) $*$is continuous ;
(iii) $a * 1 = a \ \forall \ a \epsilon [0,1]$
(iv) $a_1 * b_1 \le a_2 * b_2$ whenever $a_1 \le a_2 , b_1 \le b_2$ $\forall a_1, b_1, a_2 , b_2 \in [0,1]$.

Definition 2.2[15] A binary operation$\diamond$: $[0,1] \times [0,1] \to [0,1]$ is continuous t-co norm if $\diamond$ satisfies the following conditions:

(i) $\diamond$ is commutative and associative;
(ii) $\diamond$is continuous ;
(iii) $a \diamond 0 = a \ \forall \ a \epsilon [0,1]$
(iv) $a_1 \diamond b_1 \le a_2 \diamond b_2$ whenever $a_1 \le a_2 , b_1 \le b_2$ $\forall a_1, b_1, a_2 , b_2 \in [0,1]$.

Definition 2.3[6]. A 5-tuple $(X, M, N, *, \diamond)$ is called a non Archimedean *intuitionistic fuzzy metric space* if X is an arbitrary set, $*$ is a continuous t-norm, $\diamond$ is continuous t-conorm and M, N are intuitionistic fuzzy sets on $X^2 \times [0,\infty)$ satisfying the following conditions

(i)$M(x,y,t) + N(x,y,t) \le 1 \ \ \forall x, y \ \epsilon X \ and \ t > 0.$
(ii)$M(x,y,0) = 0 \ \forall x, y \ \epsilon X$
(iii)$M(x,y,t) = 1, \forall x, y \ \epsilon X \ \ and \ t > 0$ iff $x = y$
(iv)$M(x,y,t) = M(y,x,t) \ \forall x, y \ \epsilon X \ \ and \ t > 0$
(v)$M(x,y,\max\{t_1,t_2\}) \ge \ M(x,y,t_1) * M(y,z,t_2)$ $\forall x, y, z \ \epsilon X \ and \ t_1, t_2 > 0$
(vi)$M(x\ ,y,.) : \ [0,\infty) \to \ [0,1]$ is left continuous $\forall x, y \ \epsilon X$
(vii) $\lim_{t\to\infty} M(x,y,t) \ = \ 1$
(viii) $N(x,y,0) \ = \ 1$
(ix) $N(x,y,t) = 0, \forall x, y \ \epsilon X \ \ and \ t > 0$ iff $x = y$
(x)$N(x,y,t) = N(y,x,t) \ \forall x, y \ \epsilon X \ \ and \ t > 0$
(xi)$N(x,y,\min\{t_1,t_2\}) \le \ N(x,y,t_1) \diamond N(y,z,t_2)$ $\forall x, y, z \ \epsilon X \ and \ t_1, t_2 > 0$
(xii) $N(x\ ,y,\diamond) : \ [0,\infty) \to \ [0,1]$ is right continuous
(xiii) $\lim_{t\to\infty} N(x,y,t) \ = \ 0.$

Then $(M$,N) is called an *intuitionistic fuzzy metric space* on X.The functions $M(x,y,t)$ and $N(x,y,t)$ denote the degree of nearness and the degree of non nearness between x and y with respect to t respectively.

Definition 2.4[7].Let$(X, M, N, *, \diamond)$ be a non Archimedean *intuitionistic fuzzy metric space* then a sequence $\{x_n\}$ in X is said to be

(i)Convergent to a point $x \in X$ if $\lim_{n\to\infty} M(x_n, x; t) = 1$ and $\lim_{n\to\infty} N(x_n, x; t) = 0 \quad \forall\ t > 0$.

(ii) Cauchy sequence if $\lim_{n\to\infty} M(x_{n+p}, x_n; t) = 1$ and $\lim_{n\to\infty} N(x_{n+p}, x_n; t) = 0 \quad \forall\, t > 0$ and $p > 0$.

Definition 2.5 Let $(X, M, N, *, \diamond)$ be a non Archimedean *intuitionistic fuzzy metric space* then it is said to be complete if and only if every Cauchy sequence is convergent in X .

Definition 2.6.A function M is continuous in non Archimedean *intuitionistic fuzzy metric space* iff whenever $x_n \to x, y_n \to y$ then $\lim_{n\to\infty} M(x_n, y_n, t) = M(x, y, t)$ and $\lim_{n\to\infty} N(x_n, y_n, t) = N(x, y, t) \quad \forall\, t > 0$.

Definition 2.7[6]. A binary operation$*: [0,1] \times [0,1] \times [0,1] \to [0,1]$ is continuous t-norm if $*$ satisfies the following conditions:

(i)$*$ is commutative and associative;
(ii) $*$is continuous ;
(iii)$a * 1 = a\ \forall\ a \epsilon [0,1]$
(iv) $a_1 * b_1 * c_1 \le a_2 * b_2 * c_2$ whenever $a_1 \le a_2, b_1 \le b_2, c_1 \le c_2$
$\forall a_1, b_1, c_1, a_2, b_2, c_2 \in [0,1]$.

Definition 2.8[6]. A binary operation$\diamond: [0,1] \times [0,1] \times [0,1] \to [0,1]$ is continuous t-conorm if$\diamond$ satisfies the following conditions:

(i)$\diamond$ is commutative and associative;
(ii)$\diamond$is continuous ;
(iii)$a \diamond 0 = a\ \forall\ a \epsilon [0,1]$
(iv) $a_1 \diamond b_1 \diamond c_1 \le a_2 \diamond b_2 \diamond c_2$ whenever $a_1 \le a_2, b_1 \le b_2, c_1 \le c_2$
$\forall a_1, b_1, c_1, a_2, b_2, c_2 \in [0,1]$.

Definition 2.9[6]. A 5-tuple $(X, M, N, *, \diamond)$ is called a non Archimedean *intuitionistic fuzzy 2 metric space* if X is an arbitrary set, $*$ is a continuous t-norm, $\diamond$ is continuous t-conorm and M, N are intuitionistic fuzzy sets on $X^3 \times [0, \infty)$ satisfying the following conditions $\forall\, x, y, z, u \in X$ and $t_1, t_2, t_3 > 0$

(i)$M(x, y, z, t) + N(x, y, z, t) \le 1$
(ii)$M(x, y, z; 0) = 0$
(iii)$M(x, y, z; t) = 1, t > 0$ and when at least two of the three points are equal (iv)$M(x, y, z; t) = M(x, z, y; t) = M(z, y, x; t)$
(v)$M(x, y, z; \max\{t_1, t_2, t_3\}) \ge M(x, y, u; t_1) * M(x, u, z; t_2) * M(u, y, z; t_3)$
(vi)$M(x\ , y, z; *) :\ [0, \infty) \to\ [0,1]$ is left continuous.
(vii) $\lim_{t\to\infty} M(x, y, z;\ t)\ =\ 1$
(viii) $N(x, y, z;\ 0)\ =\ 1$
(ix) $N(x, y, z; t) = 0, t > 0$ and when at least two of the three points are equal (x)$N(x, y, z; t) = N(x, z, y; t) = N(z, y, x; t)$
(xi)$N(x, y, z; \min\{t_1, t_2, t_3\}) \le\ N(x, y, u; t_1) \diamond N(x, u, z; t_2) \diamond N(u, y, z; t_3)$
(xii) $N(x\ , y, z; \diamond) :\ [0, \infty) \to\ [0,1]$ is right continuous
(xiii) $\lim_{t\to\infty} N(x, y, z;\ t)\ =\ 0$.

Definition 2.10[7].Let $(X, M, N, *, \diamond)$ be a non Archimedean *intuitionistic fuzzy 2 metric space* then
a sequence $\{x_n\}$ in X is said to be

(i)Convergent to a point $x \in X$ if $\lim_{n\to\infty} M(x_n, x, a; t) = 1$ and $\lim_{n\to\infty} N(x_n, x, a; t) = 0 \quad \forall\ a \in X, t > 0$.

(ii) Cauchy sequence if $\lim_{n\to\infty} M(x_{n+p}, x_n, a; t) = 1$ and $\lim_{n\to\infty} N(x_{n+p}, x_n, a; t) = 0 \quad \forall\ a \in X, t > 0$ and $p > 0$.

Definition 2.11[13] Let $(X, M, N, *, \diamond)$ be a non Archimedean *intuitionistic fuzzy 2 metric space* then it is said to be complete if and only if every Cauchy sequence is convergent in X .

Definition 2.12[13] A function M is continuous in non Archimedean *intuitionistic fuzzy 2 metric space* iff whenever $x_n \to x, y_n \to y$ then $\lim_{n\to\infty} M(x_n, y_n, a; t) = M(x, y, a; t)$ and $\lim_{n\to\infty} N(x_n, y_n, a; t) = N(x, y, a; t) \quad \forall a \in X and\ \forall\ t > 0$.

Definition 2.13[22] A binary operation$*: [0,1] \times [0,1] \times [0,1] \times [0,1] \to [0,1]$ is continuous t-norm if $*$ satisfies the following conditions:

(i)$*$ is commutative and associative;
(ii) $*$is continuous ;
(iii)$a * 1 = a\ \forall\ a \epsilon [0,1]$
(iv) $a_1 * b_1 * c_1 * d_1 \le a_2 * b_2 * c_2 * d_2$ whenever $a_1 \le a_2, b_1 \le b_2, c_1 \le c_2, d_1 \le d_2$
$\forall a_1, b_1, c_1, d_1, a_2, b_2, c_2, d_2 \in [0,1]$.

Definition2.14[22]A binary operation$\diamond: [0,1] \times [0,1] \times [0,1] \times [0,1] \to [0,1]$ is continuous t-conorm if$\diamond$ satisfies the following conditions:

(i)$\diamond$ is commutative and associative;

(ii)$\diamond$is continuous ;

(iii)$a \diamond 0 = a \; \forall \; a\epsilon[0,1]$

(iv) $a_1 \diamond b_1 \diamond c_1 \diamond d_1 \leq a_2 \diamond b_2 \diamond c_2 \diamond d_2$ whenever $a_1 \leq a_2 , b_1 \leq b_2 , c_1 \leq c_2, d_1 \leq d_2$

$\forall a_1, b_1, c_1, d_1, a_2 , b_2 , c_2, d_2 \in [0,1]$.

Definition 2.15[6]. A 5-tuple $(X, M, N,*,\diamond)$ is called a non Archimedean *intuitionistic fuzzy 3 metric space* if X is an arbitrary set, $*$ is a continuous t-norm, $\diamond$ is continuous t-conorm and M, N are intuitionistic fuzzy sets on $X^4 \times [0,\infty)$ satisfying the following conditions $\forall \; x, y, z, w, u \in X$ and $t_1, t_2, t_3, t_4 > 0$

(i)$M(x, y, z, w, t) + N(x, y, z, w, t) \leq 1$

(ii)$M(x, y, z, w; 0) = 0$

(iii)$M(x, y, z, w; t) = 1, t > 0$ [Only when the three simplex (x, y, z, w)degenrate]

(iv)$M(x, y, z; t) = M(x, w, z, y; t) = M(y, z, w, x; t) = M(z, w, x, y; t) = \cdots$

(v) $M(x, y, z, w; \max\{t_1, t_2, t_3\}) \geq M(x, y, z, u; t_1) * M(x, y, u, w; t_2) * M(x, u, z, w; t_3) * M(u, y, z, w; t_4)$

(vi)$M(x , y, z, w;*) : [0,\infty) \to [0,1]$ is left continuous.

(vii) $\lim_{t\to\infty} M(x, y, z, w; t) = 1$

(viii)$N(x, y, z, w; 1) = 1$

(ix)$N(x, y, z, w; t) = 0, t > 0$ [Only when the three simplex (x, y, z, w)degenrate]

(x)$N(x, y, z; t) = N(x, w, z, y; t) = N(y, z, w, x; t) = N(z, w, x, y; t) = \cdots$

(v) $N(x, y, z, w; \min\{t_1, t_2, t_3\}) \leq N(x, y, z, u; t_1) \diamond N(x, y, u, w; t_2) \diamond N(x, u, z, w; t_3) \diamond N(u, y, z, w; t_4)$

(vi)$N(x , y, z, w;\diamond) : [0,\infty) \to [0,1]$ is right continuous.

(vii) $\lim_{t\to\infty} N(x, y, z, w; t) = 0$

Definition 2.16[22]. Let $(X, M, N,*,\diamond)$ be a non Archimedean intuitionistic fuzzy 3 metric space then

A sequence $\{x_n\}$ in X is said to be

(i)Convergent to a point $x \in X$ if

$\lim_{n\to\infty} M(x_n, x, a, b; t) = 1$ and $\lim_{n\to\infty} N(x_n, x, a, b; t) = 0 \quad \forall \; a, b \in X, t > 0$.

(ii) Cauchy sequence if

$\lim_{n\to\infty} M\big(x_{n+p}, x_n, a, b; t\big) = 1$ and $\lim_{n\to\infty} N\big(x_{n+p}, x_n, a, b; t\big) = 0 \quad \forall \; a \in X, t > 0$ and $p > 0$.

Definition 2.11[22]Let $(X, M, N,*,\diamond)$ be a non Archimedean *intuitionistic fuzzy 3 metric space* then it is said to be complete if and only if every Cauchy sequence is convergent in X .

Definition 2.12[22]A function M is continuous in non Archimedean *intuitionistic fuzzy 3 metric space* iff whenever $x_n \to x, y_n \to y$ then $\lim_{n\to\infty} M(x_n, y_n, a, b; t) = M(x, y, a, b; t)$ and $\lim_{n\to\infty} N(x_n, y_n, a, b; t) = N(x, y, a, b; t)$ $\forall a, b \in X$ *and* $\forall \; t > 0$.

3. Results and Discussion

The following theorem is Jungck's [8] generalization of the contraction principle for metric spaces

Theorem 1. Let f be a continuous mapping of a complete metric space (X, d) into itself and $g: X \to X$ be a mapping .If

(i) $g\ (X) \subset f\ (X)$

(ii) g commutes with f

(iii) $d(g(x) , g(y)) \leq \alpha d(f\ (x), f\ (y))$for some $\alpha \in (0,1)$ and for all $x , y \in X$. Then f

and g have a unique common fixed point.

Now we give the analogue of the above theorem in non Archimedean *intuitionistic fuzzy 2 metric space* .

Theorem 2 . Let $(X, M, N,*,\diamond)$ be a complete non Archimedean *intuitionistic fuzzy 2 metric space* and $f, g: X \to X$ be a mapping satisfying;

(i) $g\ (X) \subset f\ (X)$

(ii) f is continuous

(iii) $M(g(x), g(y), a; \alpha t) \geq M(f\ (x), f\ (y), a; t) \quad \forall \; x , y \in X, 0 < \alpha < 1$

and $\lim_t M(x, y, a; t) = 1$

$N(g(x), g(y), a; \alpha t) \leq N(f\ (x), f\ (y), a; t) \quad \forall \; x , y \in X, 0 < \alpha < 1$

and $\lim_t N(x, y, a; t) = 0$

Then $f$and g have a unique common fixed point..

Proof . .Let $x_0 \in X$.then we can find x_1 such that $f(x_1) = g(x_0)$. By induction, we can define a sequence $\{x_n\}$ in X such that $f(x_n) = g(x_{n-1})$.

Taking $x = x_n$ and $y = x_{n-1}$ in (iii) , we have

$$M(f(x_n), f(x_{n-1}), a; t) = M(g(x_n), g(x_{n-1}), a; t)$$

$$\geq M\left(f(x_{n-1}), f(x_n), a; \frac{t}{\alpha}\right)$$

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$$\geq M\left(f(x_0), f(x_1), a; \frac{t}{\alpha^n}\right)$$

So for any positive integer p,

$M(f(x_n), f(x_{n+1}), a; t) \geq \ M\left(f(x_0), f(x_p), f(x_1); \frac{t}{3\alpha^n}\right) *$
$M\left(f(x_0), f(x_1), a; \frac{t}{\alpha^n}\right) *$
$M\left(f(x_1), f(x_p), a; \frac{t}{\alpha^n}\right)$

Because $\lim_t M\left(f(x_0), f(x_1), a; \frac{t}{\alpha^n}\right)$=1, it follows that $M(f(x_n), f(x_{n+1}), a; t) \geq 1 * 1 * 1 \geq 1$

Similarly $N(f(x_n), f(x_{n+1}), a; t) = N(g(x_n), g(x_{n-1}), a; t)$

$\leq \ N\left(f(x_{n-1}), f(x_n), a; \frac{t}{\alpha}\right)$
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$\leq \ N\left(f(x_0), f(x_1), a; \frac{t}{\alpha^n}\right)$

So for any positive integer p,

$N(f(x_n), f(x_{n+1}), a; t) \leq \ N\left(f(x_0), f(x_p), f(x_1); \frac{t}{3\alpha^n}\right) \diamond$
$N\left(f(x_0), f(x_1), a; \frac{t}{\alpha^n}\right) \diamond$
$N\left(f(x_1), f(x_p), a; \frac{t}{\alpha^n}\right)$

Because $\lim_t N\left(f(x_0), f(x_1), a; \frac{t}{\alpha^n}\right) = 0$, it follows that $N(f(x_n), f(x_{n+1}), a; t) \leq 0 \diamond 0 \diamond 0 \leq 0$.

Thus $\{f(x_n)\}$ is a Cauchy sequence and by the completeness of space X, $\{f(x_n)\}$ converges to y.So $\{g(x_{n-1})\} = \{f(x_n)\}$ converges to y.Also the continuity of f implies the continuity of g.

So $g(f(x_n))$ converges to $g(y)$ .However $g(f(x_n)) = f(g(x_n))$ by the commutativity of f and g .Thus $f(g(x_n)) \to g(y)$.But $f(g(x_n)) \to f(y)$. .Because the limits are unique, So $f(y) = g(y)$ and $f(g(y)) = f(f(y))$.

Now again using (iii)

$M(g(y), g(g(y)), a; t) \geq \ M\left(f(y), f(g(y)), a; \frac{t}{\alpha}\right)$
$= M\left(g(y), g(g(y)), a; \frac{t}{\alpha}\right)$
$\geq M\left(g(y), g(g(y)), a; \frac{t}{\alpha^2}\right)$
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$\geq M\left(g(y), g(g(y)), a; \frac{t}{\alpha^n}\right) \to 1$

$N(g(y), g(g(y)), a; t) \leq \ N\left(f(y), f(g(y)), a; \frac{t}{\alpha}\right)$
$= N\left(g(y), g(g(y)), a; \frac{t}{\alpha}\right)$
$\leq N\left(g(y), g(g(y)), a; \frac{t}{\alpha^2}\right)$
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$\leq N\left(g(y), g(g(y)), a; \frac{t}{\alpha^n}\right) \to 0$

Thus $g(y) = g(g(y)) = f(g(y))$. So $g(y)$ is a common fixed point of f and g.

If y and z are two fixed points common to f and g, then

$1 \geq M(y, z, a; t) = M(g(y), g(z), a; t)$
$\geq M(f(y), f(z), a; \frac{t}{\alpha})$
$= M(y, z, a; \frac{t}{\alpha})$
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$\geq M(y, z, a; \frac{t}{\alpha^n}) \to 1$

Similarly $0 \leq N(y, z, a; t) = N(g(y), g(z), a; t)$
$\geq N(f(y), f(z), a; \frac{t}{\alpha})$
$= N(y, z, a; \frac{t}{\alpha})$
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$\leq N(y, z, a; \frac{t}{\alpha^n}) \to 0$

So $y = z$.

Theorem 3 . Let $(X, M, N, *, \diamond)$ be a complete non Archimedean *intuitionistic fuzzy 3 metric space* and $f, g: X \to X$ be a mapping satisfying;

(i) $g(X) \subset f(X)$

(ii) f is continuous

(iii) $M(g(x), g(y), a, b; \alpha t) \geq M(f(x), f(y), a, b; t)$ $\forall$ $x, y \in X, 0 < \alpha < 1$
and $\lim_t M(x, y, a, b; t) = 1$
$N(g(x), g(y), a, b; \alpha t) \leq N(f(x), f(y), a, b; t)$ $\forall$ $x, y \in X, 0 < \alpha < 1$
and $\lim_t N(x, y, a, b; t) = 0$

Then $f$and g have a unique common fixed point..

Proof . .Let $x_0 \in X$.then we can find x_1 such that $f(x_1) = g(x_0)$. By induction, we can define a sequence $\{x_n\}$ in X such that $f(x_n) = g(x_{n-1})$.

Taking $x = x_n$ and $y = x_{n-1}$ in (iii) , we have

$M(f(x_n), f(x_{n-1}), a, b; t) = M(g(x_n), g(x_{n-1}), a, b; t)$

$\geq M\left(f(x_{n-1}), f(x_n), a, b; \frac{t}{\alpha}\right)$
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$\geq M\left(f(x_0), f(x_1), a, b; \frac{t}{\alpha^n}\right)$

So for any positive integer p,

$M(f(x_n), f(x_{n+1}), a, b; t) \geq \ M\left(f(x_0), f(x_p), f(x_1); \frac{t}{3\alpha^n}\right) *$

$M\left(f(x_0), f(x_1), a, b; \frac{t}{\alpha^n}\right) *$

$$M\left(f(x_1), f(x_p), a, b; \frac{t}{\alpha^n}\right)$$

Because $\lim_t M\left(f(x_0), f(x_1), a, b; \frac{t}{\alpha^n}\right)=1$, it follows that $M(f(x_n), f(x_{n+1}), a, b; t) \geq 1 * 1 * 1 \geq 1$

Similarly $N(f(x_n), f(x_{n+1}), a, b; t) = N(g(x_n), g(x_{n-1}), a, b; t)$

$$\leq N\left(f(x_{n-1}), f(x_n), a, b; \frac{t}{\alpha}\right)$$

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$$\leq N\left(f(x_0), f(x_1), a, b; \frac{t}{\alpha^n}\right)$$

So for any positive integer p,

$N(f(x_n), f(x_{n+1}), a, b; t) \leq$

$$N\left(f(x_0), f(x_p), f(x_1); \frac{t}{3\alpha^n}\right) \diamond N\left(f(x_0), f(x_1), a, b; \frac{t}{\alpha^n}\right) \diamond$$

$$N\left(f(x_1), f(x_p), a, b; \frac{t}{\alpha^n}\right)$$

Because $\lim_t N\left(f(x_0), f(x_1), a, b; \frac{t}{\alpha^n}\right) = 0$, it follows that $N(f(x_n), f(x_{n+1}), a, b; t) \leq 0 \diamond 0 \diamond 0 \leq 0$.

Thus $\{f(x_n)\}$ is a Cauchy sequence and by the completeness of space X, $\{f(x_n)\}$ converges to y.So $\{g(x_{n-1})\} = \{f(x_n)\}$ converges to y.Also the continuity of f implies the continuity of g.

So $g(f(x_n))$ converges to $g(y)$ .However $g(f(x_n)) = f(g(x_n))$ by the commutativity of f and g .Thus $f(g(x_n)) \to g(y)$.But $f(g(x_n)) \to f(y)$. .Because the limits are unique, So $f(y) = g(y)$ and $f(g(y)) = f(f(y))$.

Now again using (iii)

$$M(g(y), g(g(y)), a, b; t) \geq M\left(f(y), f(g(y)), a, b; \frac{t}{\alpha}\right)$$

$$= M\left(g(y), g(g(y)), a, b; \frac{t}{\alpha}\right)$$

$$\geq M\left(g(y), g(g(y)), a, b; \frac{t}{\alpha^2}\right)$$

………………………….

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$$\geq M\left(g(y), g(g(y)), a, b; \frac{t}{\alpha^n}\right) \to 1$$

$$N(g(y), g(g(y)), a, b; t) \leq N\left(f(y), f(g(y)), a, b; \frac{t}{\alpha}\right)$$

$$= N\left(g(y), g(g(y)), a, b; \frac{t}{\alpha}\right)$$

$$\leq N\left(g(y), g(g(y)), a, b; \frac{t}{\alpha^2}\right)$$

………………………….

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$$\leq N\left(g(y), g(g(y)), a, b; \frac{t}{\alpha^n}\right) \to 0$$

Thus $g(y) = g(g(y)) = f(g(y))$. So $g(y)$ is a common fixed point of f and g.

If y and z are two fixed points common to f and g, then

$$1 \geq M(y, z, a, b; t) = M(g(y), g(z), a, b; t)$$

$$\geq M(f(y), f(z), a, b; \tfrac{t}{\alpha})$$

$$= M(y, z, a, b; \tfrac{t}{\alpha})$$

……………..

……………..

$$\geq M(y, z, a, b; \tfrac{t}{\alpha^n}) \to 1$$

Similarly $0 \leq N(y, z, a, b; t) = N(g(y), g(z), a, b; t)$

$$\geq N(f(y), f(z), a, b; \tfrac{t}{\alpha})$$

$$= N(y, z, a, b; \tfrac{t}{\alpha})$$

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$$\leq N(y, z, a, b; \tfrac{t}{\alpha^n}) \to 0$$

So $y = z$.

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